DSPA, 2004, V.1, pp. 42-44.

DESIGN OF MULTIPLIERLESS HALF-BAND DIGITAL FILTERS BASED ON ALLPASS NETWORKS

Mingazin A.
RADIS Ltd
а/я 20, 124460, Moscow, Zelenograd, Russia,
Tel/Fax (095)535-02-70, 532-06-63 e-mail: alexmin@orc.ru

Abstract. A simplified approach of variation of initial parameters can be applied for the design of multiplierless half-band digital filters based on parallel connection of two cascaded allpass networks. An example of 9-th order filter design is discussed. A catalog of 15 Zolotarev-Cauer 3-rd order half-band filters using no more than two adders instead of multipliers is presented.

1. Introduction. Half-band digital filters are widely used at interpolation and decimation in many digital signal processing systems. In particular they can be used as a basis for narrow-band filters, subband codecs, transmultiplexers and other devices. The multiplierless half-band digital filters based on parallel connection of two allpass networks using cascaded 2-nd order sections are very economic for an implementation on custom or semi-custom VLSI. In such filters half of coefficients are equal to zero, and each nonzero coefficient is a combination of numbers equal to powers of two. In this case multiplication on single coefficient in each section is replaced on shift and adding (or subtraction) operations. It is important to minimize the total number of the adders replacing multipliers [1-5]. Two stage algorithm of variation of initial parameters (VIP) was applied for this purpose in [4] and its efficiency has been confirmed on particular examples. In this paper the design problem of multiplierless half-band digital filters is adjusted and more simple for programming VIP algorithm which leads to the same results as the algorithm in [4] is offered to use.

2. Zolotarev-Cauer half-band filters. The half-band digital filters based on parallel connection of two allpass networks is described by a transfer function:

$$H(z) = \frac{1}{2}\Big(\prod_{i=1,3,...}^{K} \frac{\beta_i + z^{-2}}{1+\beta_i z^{-2}} \pm z^{-1} \prod_{i=2,4,...}^{K} \frac{\beta_i + z^{-2}}{1+\beta_i z^{-2}} \Big), \tag{1}$$

$$K \le (N-1)/2, \quad 0 < \beta_1 < \beta_2 < ... < \beta_K < 1.$$

Here N is the odd filter order, the sign plus and minus corresponds of the low- and high-pass filter respectively.

For Zolotarev-Cauer half-band filters at fixed N the coefficients in (1) are some functions of the only initial parameter :

$$\beta_i = \Psi_i(f_s), \qquad i = 1,2,3,...,(N-1)/2 \tag{2}$$

where f_s is the stopband edge normalized relation to a sampling frequency.

The half-band 3-rd order filter is described by the only coefficient β_1. Using the explicit equations for (2) from [6], it can be shown that at the change f_s in an interval $0.25 < f_s < 0.5$ (for low-pass filters) the value β_1 monotonously changes in the range $1 > \beta_1 > 1/3$. Any value of the coefficient, including a quantized value, from this range corresponds to a Zolotarev-Cauer half-band filter. It is interesting that $\beta_1 = 1/3$ determines the Butterworth 3-rd order half-band filter and therefore there are no such filters with a quantized coefficient (it is known that coefficients of Butterworth half-band filters depends on N only).

3. Problem of multiplierless half-band filter design. For a half-band filter design with the transfer function (1) it is sufficiently to control either the stopband or passband. If the coefficients in (1) are determined as in (2) then multiplierless half-band filter design problem by a VIP algorithm can be formulated as

$$\Sigma_m(f_s) \to \min_{f_s}, \ \tilde{a}_0(f_s) \ge a_{0\min}, \ f_s \in S$$

where $\tilde{a}_0$ is the minimum attenuation of the magnitude response in the nominal stopband; the symbol ~ means adequacy to the quantized coefficients in (2); $a_{0\min}$ is the given tolerable attenuation; Σ_m - the total number of adders, replacing multipliers; S is the space of possible values f_s which includes the given nominal value f_{sn}.

As a result of the design we shall obtain quantized coefficients which will not correspond to a Zolotarev-Cauer filter except N=3 and $1>\beta_1>1/3$. Notice, the multiplierless Zolotarev-Cauer half-band filter design at N=3 can be regarded as a trivial problem which can be solved without the use of (2) by direct substitution of "good" values β_1 in (1) with the subsequent analysis of $\tilde{a}_0$.

4. Design algorithm. A decision of the formulated problem can be found by the two step VIP algorithm from [4]. The first stage is a definition of starting points by a branch and bound method, the second stage is a local variation of the parameter f_s. In the algorithm a transcendental equation is repeatedly solved. Simplified VIP algorithm, in essence similar described in [7], is free from this procedure. It consists in the following: the parameter f_s changes in the space S for a set of values of the coefficient quantization step $q = q_{max}, q_{max}/2, \ldots$ until for some $q = q_0$ and $q_0/2$ all tolerable solutions with $\tilde{a}_0 \geq a_{0\min}$ will be found. Further a variant with the minimal number Σ_m should be chosen.

Notice that in S there are the finite number of subspaces and the own coefficient vector corresponds to each of these subspaces [7]. In the algorithm all subspaces are found and for each subspace the evaluation of a solution on tolerability is carried out only once. In a heuristic algorithm [2] these details are not described.

The results of a half-band filter design from [2] and [4] can be reproduced by the simplified VIP algorithm. Further other filter design example is discussed and a catalogue of Zolotarev-Cauer 3-rd order half-band filters which has been obtained by solving (2) for a set of quantized coefficients is presented.

5. Example. Half-band filter specifications: $a_{0\min} = 47$дБ, f_{sn}=0.27, N=9.
Authors [5] have found by a variation of coefficients (VC) the following solution:

$$\Sigma_m=7,\ \beta_1 = 2^{-3} + 2^{-6},\ \beta_2 = 2^{-1} - 2^{-4} - 2^{-9},\ \beta_3 = (1-2^{-2})(1-2^{-4}),\ \beta_4 = 1 - 2^{-3} + 2^{-5}.$$

It is possible to check up that these coefficients corresponds to $\tilde{a}_0$=45 dB instead of 47 dB.

The described above VIP algorithm leads to a set of solutions with $\tilde{a}_0 \geq$ 45 dB and two of them are

var.1: f_s=0.271415, Σ_m=8, $\tilde{a}_0$=47 dB,

$$\beta_1 = 2^{-3} + 2^{-7},\ \beta_2 = 2^{-2} + 2^{-3} + 2^{-5} + 2^{-7},\ \beta_3 = 2^{-1} + 2^{-3} + 2^{-4},\ \beta_4 = (1+2^{-5})(1-2^{-3}).$$

var.2: f_s = 0.265030, Σ_m=7, $\tilde{a}_0$=45 dB,

$$\beta_1 = 2^{-3} + 2^{-5},\ \beta_2 = 2^{-1} - 2^{-5} + 2^{-9},\ \beta_3 = (2^{-1} + 2^{-2})(1-2^{-6}),\ \beta_4 = 1 - 2^{-4} - 2^{-6}.$$

Var.1 satisfies to the requirement on $\tilde{a}_0$ but concedes to the solution [5] in Σ_m on one adder. Var.2 is equivalent to this solution and has other coefficients.

We use the coefficients of var.1 as initial in VC algorithm. The algorithm gives the solution:

var.3: Σ_m=6, $\tilde{a}_0$=46 dB,

$$\beta_1 = 2^{-3},\ \beta_2 = (2^{-1} - 2^{-4})(1-2^{-4}),\ \beta_3 = 2^{-1} + 2^{-3} + 2^{-4},\ \beta_4 = (1+2^{-5})(1-2^{-3}).$$

In relation to var.1 two adders are saved but $\tilde{a}_0$ is reduced on 1 dB, and in relation to var.2 and the solution [5] one adder is saved and $\tilde{a}_0$ is improved on 1 dB.

A structure of each four allpas sections in [5] contains 3 adders. Two adders is required for low- and high-pass outputs. Therefore the total number of adders in the filter structure $\Sigma = \Sigma_m + 3\times4 + 2 = 21$. If the direct form with two adders is used for realization of the allpass sections than we shall obtain Σ=16 for var.3.

6. Catalog of Zolotarev-Cauer 3-rd order half-band filters. Coefficients and parameters of Zolotarev-Cauer 3-rd order half-band digital filters are presented in the table. Here $\Delta\tilde{a}$ is the passband ripple and

Table

Filter	1	2	3	4	5	6	7	8
β_1	$2^{-1}+2^{-2}$	$2^{-1}+2^{-3}+2^{-4}$	$2^{-1}+2^{-3}$	$2^{-1}+2^{-4}$	2^{-1}	$2^{-1}-2^{-5}$	$2^{-1}-2^{-4}$	$(1-2^{-4})\times(2^{-1}-2^{-4})$
$f_s = f_{sn}$	0.263822	0.271403	0.281561	0.295354	0.314677	0.327493	0.343496	0.361343
$\tilde{a}_0$, dB	11.1	13.2	15.5	18.4	22.3	24.8	28.0	31.8
$\Delta\tilde{a}$, dB	0.35	0.22	0.12	0.063	0.026	0.014	0.0068	0.0029
$\Delta\tilde{\tau}$	5.0	3.4	2.3	1.6	1.0	0.77	0.56	0.39

continuation of the table

9	10	11	12	13	14	15
$(1+2^{-4})\times(2^{-2}+2^{-3})$	$(1-2^{-3})\times(2^{-1}-2^{-4})$	$2^{-2}+2^{-3}$	$(1-2^{-5})\times(2^{-1}-2^{-3})$	$2^{-2}+2^{-3}-2^{-6}$	$(1+2^{-3})\times(2^{-2}+2^{-4})$	$2^{-2}+2^{-4}+2^{-5}$
0.370575	0.385009	0.393448	0.408312	0.414071	0.427368	0.444521
33.9	37.3	39.5	43.7	45.5	50.1	57.3
0.0018	0.0008	0.0005	0.00018	0.0001	0.00004	0.00001
0.325	0.24	0.20	0.14	0.12	0.084	0.048

$\Delta\tilde{\tau}$ is the group delay normalized with respect to a sampling period. The number of the adding in these coefficients does not exceed two. Only for the filter-5 the multiplication on the coefficient is implemented without adders.

7. Conclusions. A simplified VIP algorithm may be applied for the design of multiplierless half-band digital filters based on parallel connection of two cascaded allpass networks. The 9-th order filter design with the minimal total number of adders replacing multipliers is discussed. It is shown that for these filter the additional reduction of the number of adders can be achieved by association the VIP algorithm and a variation of coefficients. A catalogue 15 Zolotarev-Cauer 3-rd order half-band filters which are economic for realization on VLSI is presented.

References

1. Gulluoglu S.N., Wegener W., Gazsi L. Catalog of bireciprocal wave digital low-pass filters with discrete optimized coefficients.// IASTED. 1985. June. –P. 120-123.
2. Samueli H. A low-complexity multiplierless half-band recursive digital filter design. //IEEE Trans. 1989. ASSP-37. No. 3. -P. 442- 444.
3. Milic L. D., Lutovac M. D. Design of multiplierless elliptic IIR filters with a small quantization error.//IEEE Trans. 1999. SP-47. No. 2. -P. 469- 479.
4. Mingazin A.T. Finite wordlength coefficient design of digital filters based on allpass networks.//DSPA. 1999. V. 1. -P. 117-121.
5. Lutovac B.M., Lutovac M.D. Design and VHDL description of multiplierless half-band IIR filter.//Int. J. Electron. Commun. (AEU). 2002. V.56. No.5. -P. 348-350.
6. Mingazin A.T. Computation of half-band IIR filters.//Elektrosvyaz. 1996. No. 6. -P. 36-37.
7. Mingazin A.T. Digital filter design with finite wordlength coefficients.//Elektrosvyaz. 1983. No. 7. -P. 49-53.