ANISOTROPIC LOCAL TRANSFORM BASED AVERAGE FILTERING

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Abstract. Two schemes for image denoising are presented. First one is transform-domain Wiener filtering in a moving window with averaging of the results for each pixel. Second one also relies on moving window averaging, however the Wiener filtering is substituted by anisotropic best basis search, subject to an MDL criterion.

1. Introduction

Denoising of images corrupted by additive white noise has always been a classical task in signal processing. The optimal linear filter in mean square error sense for image restoration is the Wiener filter [1]. However, it requires at least second order stationarity of the involved stochastic processes and correspondingly, knowledge of their second-order statistics. For most of the images the stationarity requirement is not satisfied, but usually the assumption of local stationarity is accurate. Thus, the moving window Wiener filter scheme with subsequent averaging as proposed in [2] is a good solution. Moreover, if the Wiener filtering operation is performed in transform domain computational complexity is decreased. Still, getting adequate estimates of the clean signal and noise variance is an open problem.

In [3] Saito suggests a scheme for simultaneous compression and denoising of a signal in transform domain utilizing the Rissanen's minimum description length (MDL) principle [4]. This principle states that from a collection of models representing the data the best model is the one giving the shortest description of the data. Saito's scheme could serve as an alternative of the local Wiener-like thresholding and allows dropping out the requirement for preliminary knowledge of signal and noise statistics.

In this paper, a denoising scheme combining the moving window averaging in DCT domain with local, MDL-based coefficient thresholding is proposed. At each position of the moving window a modification of Saito's method is implemented and then each pixel value is estimated by averaging the results for that pixel from all windows that enclose it [2]. For each window position the collection of models include anisotropic local cosine packets [5] and a search for best anisotropic local cosine basis subject to MDL cost measure is performed. The motivation behind the proposed scheme is the promising results of MDL denoising and the advantage of anisotropic transforms that allow better adaptation to the local image features.

2. Averaging Local Transform-domain Wiener Filter

The local Wiener filtering scheme is the following [2]. First, a moving window $W_{k,l}$ of size $N\times N$ is placed as to enclose pixels $(k, l), \dots ,(k+N-1, l+N-1)$. For each window position (k, l), the value of each pixel inside the window $(r_1, r_2)\in W_{k,l}$ is estimated according to the formula:

$$\hat{g}^{k,l}_{r_1,r_2} = \left(T^{-1}\left\{\eta^{k,l}_{i,j}.G^{k,l}_{i,j}\right\}\right)_{r_1,r_2}, (i,j)\in W_{k,l}, \quad (1)$$

where $T\{.\}$ and $T^{1}\{.\}$ are the forward and inverse transform respectively, η is the Wiener filter in transform domain and $G^{k,l} = T\{ g^{k,l} \}$ is the transform of the corrupted image fragment $g^{k,l}$, enclosed by the moving window $W_{k,l}$. Explicit formulas for η can be found in [2] but all of them require knowledge of the non-corrupted signal and noise statistics. They can be obtained by carefully tuned pre-filtering procedure **Ошибка! Источник ссылки не найден.**. The final estimate $\hat{g}_{r_1,r_2}$ for pixel at position (r_1, r_2) is obtained by averaging all pixel estimates $\hat{g}^{k,l}_{r_1,r_2}$ obtained from window positions $W_{k,l}$ such that $(r_1, r_2)\in W_{k,l}$:

$$\hat{g}_{r_1,r_2} = \frac{1}{N^2}\sum_{k,l:(r_1,r_2)\in W_{k,l}} \hat{g}^{k,l}_{r_1,r_2} \quad (2)$$

3. Averaging Anisotropic Local Transform Domain MDL Filtering

In the previously described scheme if we choose smaller moving window we will have better spatial resolution and the assumption of local stationarity will be better satisfied, however the overall computational complexity will increase as more moving windows will be computed. For homogenous image regions this is not necessary and better frequency resolution would be preferable. That's why we try to increase the moving window size and use inside it an anisotropic transform that better adapts to the local image features. MDL criterion is used for best basis search and also for determination of the noise suppression threshold. Again, averaging multiple pixel estimates corresponding to different window positions helps us to achieve translation invariance and to reduce Gibbs-like artifacts caused by the transform coefficient thresholding.

The anisotropic local cosine transform we use inside the moving window has the dyadic-tree structure illustrated in Figure 1 [5]. Here, each node at a level represents the projection of the image on different subspaces. Each (parent) subspace has as descendent (children) subspaces all subspaces whose box support fall into the box support of the parent subspace. More details on that structure can be found in [5]. One example of parent with left and right children subspaces is shown in Figure 2.

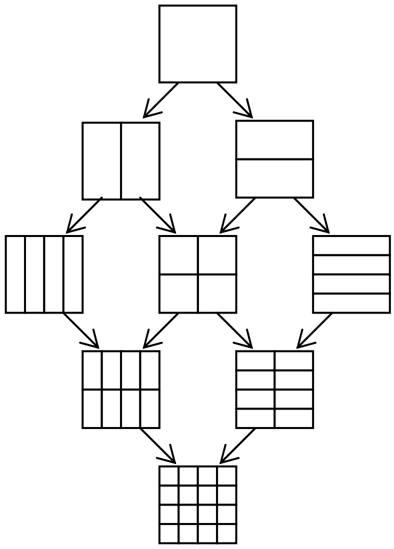

Figure 1. Tree structure used for anisotropic local cosine decomposition

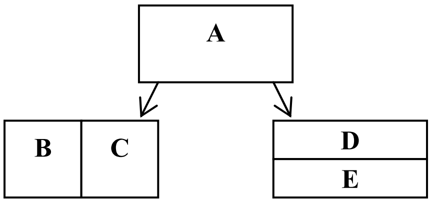

Figure 2. Parent subspace (A), left children subspaces (B and C) and right children subspaces (D and E)

The proposed algorithm is as follows:

- For each window $W_{k,l}$ ((k, l) = (0, 0), ... , (M, M), M is the size of the noisy image) decompose the corrupted image fragment $g^{k,l}$ in the tree structure shown in Figure 1.
- Starting one level up to the lowest, for each subspace (box) in each node at a level compare the MDL cost for it and its left and right children subspaces and choose the one having the smallest cost, i.e.

$A = B \cup C$, if $\min\{$ MDL($B \cup C$), MDL($D \cup E$), MDL(A) $\}$ = MDL($B \cup C$)

$A = D \cup E$, if $\min\{$ MDL($B \cup C$), MDL($D \cup E$), MDL(A) $\}$ = MDL($D \cup E$)

$A = A$, otherwise

The MDL cost for a subspace X is calculated as [3]:

$$MDL(X) = \min_{0 \le m \le n}\left(\frac{3}{2} m \log n + \frac{n}{2} \log \left\| \left(I - \Theta^{(m)} \right) V^{(X)} g^{k,l} \right\|^2 \right), \qquad (3)$$

where n are the total number of pixels in the subspace X, $V^{(X)}$ is the transform operator that projects $g^{k,l}$ on the subspace X and $\Theta^{(m)}$ is a threshold operator that keeps the m largest elements in the subspace X ($m \le n$) and sets all other to zero.

- Keep track on the separation for each subspace (box).
- Proceed across each level in the tree finishing at the topmost level. There a best anisotropic basis is found with transform projector V^* and threshold operator $\Theta^{(m^*)}$ that selects m^* largest coefficients in the root node from all N^2.
- Find estimate $\hat{g}^{k,l}$ of the "clean" signal inside the moving window $W_{k,l}$ according to the equation

$$\hat{g}^{k,l} = V^{*-1}\left(\Theta^{(m^*)} V^* g^{k,l} \right) \qquad (4)$$

- Use equation (2) to find the final estimates for all pixels by averaging.

It has to be noted that the proposed denoising algorithm can be applied with all separable wavelet transforms, not only local cosine. Furthermore if we apply it with local cosine transform the support of folding operators can be at most one pixel wide and thus the transform becomes nothing but block cosine transform. This restriction comes from the tree structure used for decomposition and the requirement that all basis functions must be orthogonal between each other.

References

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