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COMBINED TIME-FREQUENCY REGULARIZATION AND ITS APPLICATION FOR HYDROPHYSICAL MEASUREMENTS

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A broad class of signal processing problems leads to the deconvolution problem that is solved by the Fourier Transform combined with Tikhonov's regularization. This approach uses the minimal a priori information on the smoothness of the solution. Processes with small signal to noise ratio or stochastic processes, for instance, turbulence, result in large frequency oscillations, and limit the accuracy of the recovery. We use a natural additional smoothness property of the solution in the frequency domain. The possibility are appeared to generalize the regularization technique in time-frequency domains simultaneously. The suitable inversion procedure is carried out, and developed in detail for spectral analysis (SA) of stochastic wide-band stationary processes. Some results of the modified SA applications for hydrodynamic basin are described. The modified SA procedure displays a high computing efficiency, and allows to improve the SA accuracy, or, to reduce the full SA sampling at 7 - 10 times under comparable SA accuracy. It is important for on-line data processing.

Keywords: generalized regularization, spectral analysis, stochastic process, deconvolution.

1. GENERALIZED REGULARIZATION TECHNIQUE

Many problems of data processing [1-3] lead to the deconvolution problem

$$u_s(t) = K_t\, u_0(t) = \qquad , \tag{1}$$

where k(t) is the kernel of the integral system operator, $u_s(t)$, $u_0(t)$ are the measured and, respectively, unknown functions. Applying the Fourier Transform (FT) to (1) leads to the equation on frequency domain

$$U_s(\omega) = K(\omega)\, U_0(\omega). \tag{2}$$

Here $U_0(\omega)$, $U_s(\omega)$ are the FTs of the unknown and measured functions, $K(\omega)$ is a frequency response function of the dynamic system. Sometimes $K(\omega)$ can be evaluated as two FTs ratio of the measured functions (some blind systems, or the comparison method at the metrology [3,4]).

It is well-known the successful application of the usual Tikhonov regularization (where the smoothing functional is formed on time domain) to solve the deconvolution problems [1-4]. This procedure is equivalent to low-pass filtering (in the frequency domain), correlative processing, and tuning under the system operator [5]. Stochastic phenomena or processes with the small signal/noise ratio lead to additional random oscillating noise of $U_0(\omega)$ in the frequency domain. It causes the additional loss of the accuracy of the signal restoration. Usually, we cannot apply the efficient stochastic restoration procedures (Wiener, Kalman, etc) as we have not the necessary full information.

As, we generalize Tikhonov regularization on the combined time-frequency domains and develop the corresponding restoration procedure. Thus, inversion procedure has the following stages :

It is assumed the smoothness of regularizative solution, $u_\alpha(t)$ and its FT, $U_\alpha(\omega)$. Thus, we form up the smoothing functional on time and frequency domains together

$$\Phi_\alpha = 2\pi \int_{-\infty}^{\infty} \{[u_s(t) - K_t u_\alpha(t)]^2 + \alpha \sum_{m=0}^{N} \beta_m [u_\alpha^{(m)}(t)]^2\} dt +$$

$$+ \int_{-\infty}^{\infty} [|U_s - K(\omega) U_\alpha(\omega)|^2 + \alpha \sum_{m=0}^{M} k_m |U_\alpha^{(m)}(\omega)|^2] d\omega . \tag{3}$$

Here β_m, k_m are arbitrary nonegative quantities, and α is the numeral regularization parameter.

- A minimization of functional (3) leads to the regularizative solution in the frequency domain.
- The choice of optimal regularization parameter α_{op} (particular strategy is discussed) leads to the approximated solution of initial problem.
- The return in the time domain (if it need).

The proof of this procedure is analogous to Tikhonov's regularization for time domain. Proposed method uses a natural a priori information on the smoothness of the solution and its FT. Digitisation and realization of above algorithm reduce to linear algebraic band system.

1.1. Usual Tikhonov's regularization does not assume the smoothness of $U_\alpha(\omega)$, and corresponds to $k_m = 0$ in the functional (3). A minimization of (3) leads to well-known equation for $U_\alpha(\omega)$ [1,2]

$$U_\alpha(\omega)=U_s(\omega)K^*(\omega)/\{[K(\omega)K^*(\omega)-1]+P_N(\omega)\},\ P_N(\omega)=1+\alpha_0\sum_{m=0}^{N}\beta_m\omega^{2m},\ \alpha_0=\alpha/2. \quad (4)$$

Here $K^*(\omega)$ is the complex conjugate function at $K(\omega)$, and the function $P_N^{-1}(\omega)$ can be compared with the low-pass filtering function [5]. Thus, the smoothness of the regularization solution $u_\alpha(t)$ in time domain corresponds to low-pass filtering on frequency domain, correlative processing for output function $u_s(t)$, and tuning under the dynamic system operator [5].

1.2. Regularization on frequency domain assumes the solution smoothness on frequency domain (it corresponds $\beta_m = 0$). In this case a minimum of functional (3) leads to the equation for $U_\alpha(\omega)$

$$K^*(\omega)U_s(\omega)=K^*(\omega)K(\omega)U_\alpha(\omega)+\alpha_0\sum_{m=0}^{M}k_m(-1)^m U_\alpha^{(2m)}(\omega),\ \alpha_0=\alpha/2. \quad (5)$$

Here $K^*(\omega)$ is the complex conjugate function at $K(\omega)$. It can be shown the solution smoothness on frequency domain gives low- pass filtering on time domain (limitation of sampling time and the vanishing point to zero), and tuning under the system operator.

1.3. Combined time-frequency regularization corresponds the smoothness of the solution, and its FT similtaneously. A minimization of common case of (3) gives the following equation for $U_\alpha(\omega)$

$$K^*(\omega)\ U_s(\omega)\ [\ K^*(\omega)\ K(\omega) + \alpha_0 \sum_{m=0}^{N}\beta_m\omega^{2m}\ ]^{-1} = U_\alpha(\omega) + \alpha_0\sum_{m=0}^{M}k_m(-1)^m U_\alpha^{(2m)}(\omega)\quad [K^*(\omega)\ K(\omega) + \alpha_0 \sum_{m=0}^{N}\beta_m\omega^{2m}\ ]^{-1}. \quad (6)$$

Equation (6) corresponds to low-pass filtering of regularization solution in frequency and time domains together, and tunning under the system operator.

Time uniform sampling, digitisation, the FFT applying, solving of equation (6) (its discrete option), the choice of optimal regularization parameter, α_{op}, and the return in time domain by the inverse FT are the full inversion procedure for (1). Below, this technique is applied for stochastic stationary processes, where the smoothness in frequency domain is a natural one's property. Such processes are, for instance, the turbulence arised at submerged jet.

For spectral and correlative analysis of stochastic processes the choice strategy of optimal regularizative parameter will be given on next part. For general case of the equation (6), choice strategies of α_{op} will be developed on subsequent analysis.

We use the equation (6) to estimate the FT $U_s(\omega)$ of the noisy measured function, $u_s(t)$. In particular, this problem arises at the use of comparison method in metrology to calibrate the unknown sensor by the standard sensor and uniform test signal, $u_0(t)$ which acts simultaneously on both sensors. Such measuring technique leads to two integral equations :

$$u_i(t) = \int_{-\infty}^{\infty} w_i(t-\xi)\,u_0(\xi)\,d\xi, \qquad i = 1,2 \quad (7)$$

where $w_i(t)$ are impulse response for suitable sensor, $u_i(t)$ are the measurands of sensors. Applying the Fourier Transform gives the equation for the frequency response, $W_2(\omega)$ of unknown sensor :

$$W_2(\omega) = W_1(\omega)\ U_2(\omega)\ /\ U_1(\omega). \quad (8)$$

Thus we can define the frequency response of unknown sensor by means of the known frequency response, $W_1(\omega)$ of standard sensor, and the FTs of the measured functions.

The FT estimation of the measured function, $u_s(t)$ corresponds to $K(\omega)=K^*(\omega)=1$ at (1)-(6), and leads to the following equation for $U_\alpha(\omega)$:

$$U_s(\omega)P_N^{-1}(\omega)=U_\alpha(\omega)+\alpha_0 P_N^{-1}(\omega)\sum_{m=0}^{M} k_m(-1)^m U_\alpha^{(2m)}(\omega)\ ,\quad P_N^{-1}(\omega)=[1+\alpha_0\sum_{m=0}^{N}\beta_m\omega^{2m}]^{-1} \qquad (9)$$

Here $P_N^{-1}(\omega)$ is low-pass filtering function.

Optimal regularization parameter α_{op} is calculated by integral relation $||U_\alpha(\omega) - U_s(\omega)||_2^2 = 2\pi\,\delta_0^2$. Here δ_0 is a prescribed residual (this procedure is equivalent to the residual method [1,2]). We could apply formulae (9) to solve the different engineering problems, for instance, some signal recovery problems [3,4], or, to speed the spectral and correlative analysis for wide-band processes.

2. MODIFIED SPECTRAL ANALYSIS (SA) OF STOCHASTIC WIDE–BAND PROCESSES

It is very important for practice to measure and to calculate the spectrum (or spectral density) of wide-band stochastic stationary processes which are used widely on science and engineering [6-7]. The phenomenon nature and the lowest frequency of this process determine the sampling time of spectral and correlative analysis, and SA time can achive very large values. Applying of usual spectral analysis (SA) to solve this problem can cause to full sampling time (with the time average at samples) up to tens minutes. Below, we propose to use the spectrum smoothness to speed SA.

Spectrum, $S(\omega)$ of stochastic stationary process is defined as the limit $T \to \infty$ of the ensemble average [1,6,7] of FT for finite time

$$S_s(\omega) = \lim T^{-1} E\{ |U_s(\omega)|^2 \}, \qquad U_s(\omega)= \qquad , \qquad (10)$$

where E is a symbol of the ensemble average. For ergodic processes the ensemble average is replaced by the time average at samples $S(\omega) = \lim T^{-1}\, \overline{|\,U(\omega)\,|^2}$. Also, the amplitude spectrum, $|\overline{U}(\omega)| = \sqrt{S(\omega)}$ is applied widely instead of the energy spectrum $S(\omega)$.

Usual SA includes following stages to measure and to calculate the spectrum:

- the preliminary smoothing of signal by low-pass filters,
- time uniform sampling and digitisation of signal by analog/digital converter (ADC),
- the spectrum computation by FFT and the smoothing by different weight windows,
- averaging spectrum at samples.

This SA procedure for on-line processing gives small accuracy (with large random oscillations) of the spectrum recovery. Our SA experience of turbulent processes shows the SA smoothing by weight windows is inadequate.

Really, we can assume the spectrum smoothness without the additional detailed information. It permits to improve the calculated accuracy of spectrum, or to reduce the SA sampling for the same accuracy. We have two different ways to compute the modified spectrum, $S_\alpha(\omega)$.

To evaluate the modified spectrum by the first way, we can use the equations (9) with the replacement $U_s(\omega)$, $U_\alpha(\omega)$ on suitable quantities $|U_s(\omega)|$, $|U_\alpha(\omega)|$. The conclusion procedure of (9) at the limited frequency compact $[0, \omega_{max}]$ uses the additional boundary conditions

$$|U_\alpha(0)|^{(1)} = |U_\alpha(0)|^{(2)} = 0\ , \quad |U_\alpha(\omega_{max})|^{(1)} = |U_\alpha(\omega_{max})|^{(2)} = 0\ .$$

Here we use only two derivatives of functional (3), as usually it is applied. The time uniform sampling for the equation (9) gives the algebraic band system, and the band width is equal nine for this case.

Second way reduces to more economical calculated procedure. As the low-pass filtering, $P_N^{-1}(\omega)$ is conducted by electronic devices at spectral analysis, we can form the smoothing functional for the spectrum of measured function, $S_s(\omega)$, and the modified spectrum, $S_\alpha(\omega)$ in frequency domain

$$\Phi_\alpha = \int_0^{\omega_{\max}} \{[S_\alpha(\omega) - S_s(\omega)]^2 + \alpha \sum_{m=0}^{N} [S_\alpha^{(m)}(\omega)]^2\} \, d\omega \,. \qquad (11)$$

Time uniform sampling and minimization of (11) leads to linear algebraic band system with matrix, B (band width is equal (2 (N + 1)), and h is the frequency step:

$$B\,\mathbf{S_\alpha = S_s}\,,\ \mathbf{S_\alpha}=\{S_\alpha(\omega_n)\},\ \mathbf{S_s}=\{S_s(\omega_n)\},\ B=\{b_{n,m}\},\ \omega_n = n\,h\,. \qquad (12)$$

The simplest three-diagonal system gives the first order of the functional (11)

$$b_{n,n} = 1 + \gamma_1 + \alpha_0\,,\ n=1; \quad b_{n,n} = 1 + 2\,\gamma_1 + \alpha_0\,,\ n=\overline{2, n_{\max}}\,;$$
$$b_{n,n-1} = 0\,,\ n=1; \quad b_{n,n-1} = -\gamma_1\,,\ n=\overline{2, n_{\max}}\,;$$
$$b_{n,n+1} = 0,\ n=n_{\max}; \quad b_{n,n+1} = -\gamma_1\,,\ n=\overline{1, n_{\max}-1}\,; \quad \gamma_1 = \alpha_0\,k_1/h^2. \qquad (13)$$

Matrix, (13) allows to calculate the smoothing spectrum by economic iterative procedure [8].
Optimal parameter, α_{op} is calculated from an integral natural relation

$$\alpha_{op} \Rightarrow \int_0^{\infty} S_\alpha(\omega)\,d\omega = \int_0^{\infty} S_s(\omega)\,d\omega\,. \qquad (14)$$

Formulas (10)–(14) describe relations for amplitude spectrum after the replacement $S(\omega) \Rightarrow |\overline{U}(\omega)|$.

3. SA APPLICATIONS FOR HYDROPHYSICAL MEASUREMENTS.

Various Ocean investigations (physical, biological, etc) make it necessary to provide the metrology of various hydrophysical probes. Optimal way to provide the uniformity and reliability of measurements in statistical hydromechanics is the application of a “standard” turbulent model stream used as an input effect to define dynamic parameters of hydrophysical probes such as spatial and time resolutions, conversion factors, amplitude and frequency responses, etc. The use of submerged turbulent jet (for temperature and speed) in self-preservation region makes it possible.

Such standard turbulent water flow raised temperature fluctuations had been performed by free large submerged jet into hydrodynamic basin to calibrate various hydrophysical probes on wide frequency band, and to determine its amplitude-frequency responses. The calibration set-up, which uses the submerged turbulent jet after the water heating, had performed in D.I. Mendeleyev Institute of Metrology. As result, we have wide-band stochastic stationary ergodic speed and temperature signals.

Up-to-date we performed needed service and researchs to provide the calibration of temperature sensors. There are exist two problems:
- the research of random temperature fluctuations,
- the calibration of temperature sensors (determination of its amplitude-frequency response).

The modified SA procedure had been realized by the set of devices (two temperature sensors, amplifiers, low-pass filters, multiplexer, PC-card including multiplexer and ADC, PC) to measure the temperature turbulent processes into the chamber volume. Measurements and computations of amplitude spectrum of temperature fluctuations were conducted to research the set-up. Figure 1 represents the results of one measurement. Curves correspond to usual SA with different number of samples, 100 (approximately minutes 27), and the modified SA with the samples number, 100. Increasing number of samples (for usual SA) up to 800 (approximately minutes 205) leads to the same result. Thus, the application of modified SA permits to reduce samples at 8 times.

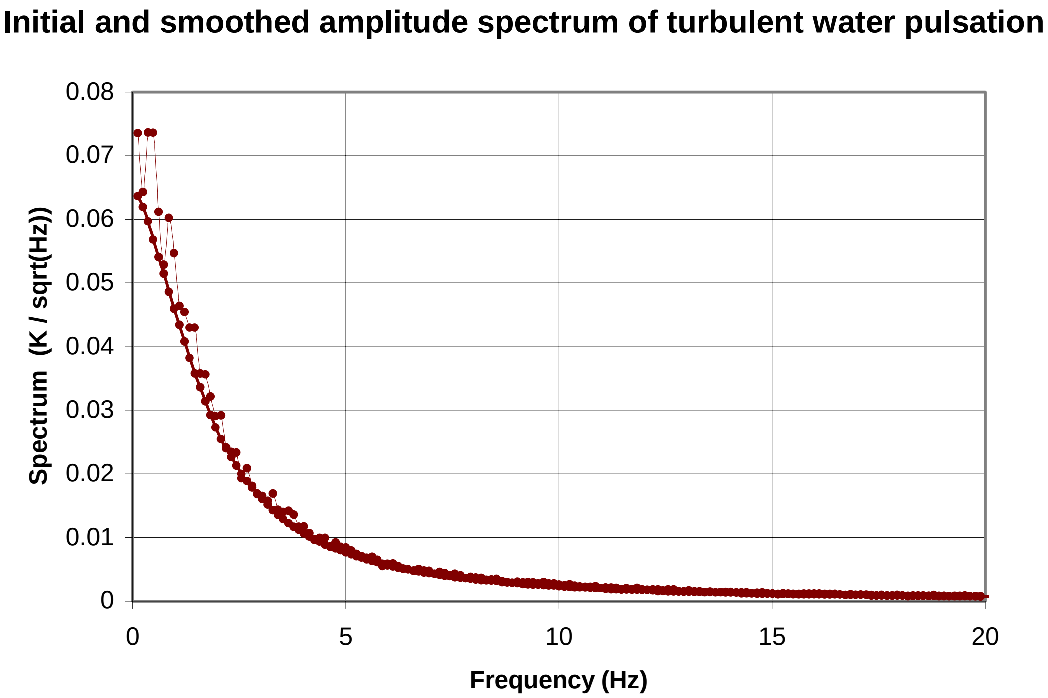

Fig. 1. Some results of usual and modified application for amplitude spectrum of turbulent water temperature pulsation in hydrophysical calibration set-up.

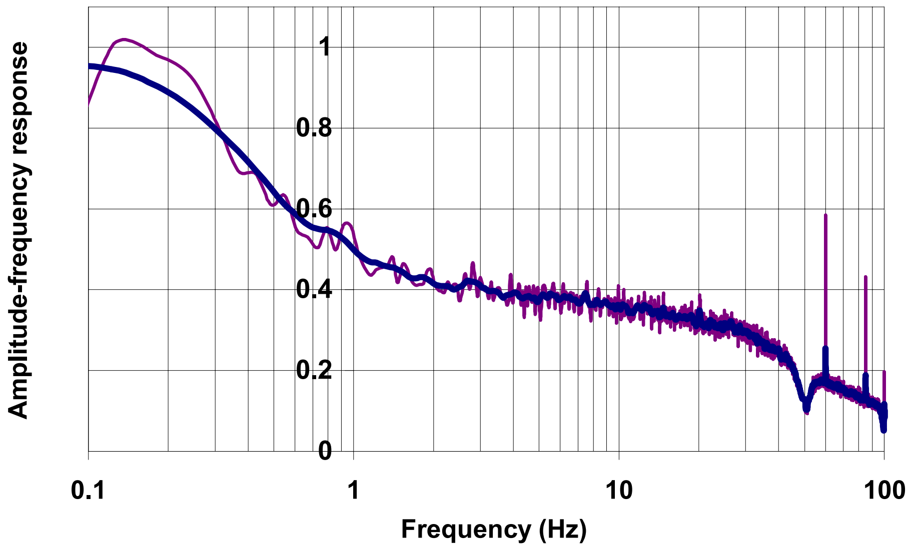

Fig. 2 . Comparison between usual and modified SA for computing of amplitude-frequency response of the temperature sensor versus frequency.

The second problem for the hydrophysical set-up is the calibration of temperature sensors (computation of amplitude-frequency response). Instead of formulae (8), we can calculate amplitude-frequency response of sensor by the relation:

$$|W_2(\omega)| = |W_1(\omega)| \quad S_2(\omega) / S_1(\omega) \tag{15}$$

Figure 2 displays the calibration result of some temperature sensor determined by various techniques (with and without the smoothing of the formulae (15)).

CONCLUSIONS.

Results of measurements and computing proved the reduction of the measurement uncertainty (accuracy) at 3-4 time for the calculating spectrum on the same sampling, or, the reduction of full SA time (at 8-10 times) at the comparable SA accuracy.

The combined time-frequency inversion displays high computing efficiency of data processing for wide-band stochastic processes as turbulence. We wait this technique can be applied efficiently to solve different engineering and science problems described as the deconvolution problems.

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