PERFECT OBSERVERS FOR SINGULAR 2D ROESSER MODEL

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Abstract. A new concept of the perfect singular observer for singular 2D Roesser model is proposed. Necessary and sufficient conditions are established for the existence of the perfect observer for singular 2D Roesser model. A procedure for design of the perfect observer is also derived.

Key Words: existence, design, perfect observer, singular 2D Roesser model

1. Introduction.

The most popular models of two-dimensional (2D) systems are the models introduced by Roesser [11], Fornasini and Marchesini [3,4] and Kurek [10]. The models have been generalised for singular systems in [7,6]. The existence and design methods of the observers for singular 1D linear systems have been considered in many papers and books [1,2,6,8,12-14]. Dai has shown [2] that it is possible to construct a singular observer which reconstruct exactly the state vector $x(k)$ of the singular system $Ex(k+1) = Ax(k) + Bu(k)$ for all $k = 0,1,...$

The main subject of this short paper is to extend the concept of perfect observer for singular 2D Roesser model.

Necessary and sufficient conditions will be established for the existence of the perfect observer for singular 2D Roesser model and a procedure for design of this perfect observer will be derived.

2. Preliminaries and problem formulation.

Let $R^{n\times m}$ be the set of $n\times m$ real matrices and $R^n := R^{n\times 1}$. The set of non-negative integers will be denoted by Z_+.

Consider the singular 2D Roesser model [11,6]

(1a) $$E\begin{bmatrix} x^h_{i+1,j} \\ x^v_{i,j+1} \end{bmatrix} = A\begin{bmatrix} x^h_{ij} \\ x^v_{ij} \end{bmatrix} + Bu_{ij}$$

(1b) $$y_{ij} = C\begin{bmatrix} x^h_{ij} \\ x^v_{ij} \end{bmatrix} \quad , i,j \in Z_+$$

where $x^h_{ij} \in R^{n_1}$ and $x^v_{ij} \in R^{n_2}$ are the horizontal and vertical semi-state vectors ate the point (i,j), $u_{ij} \in R^m$ is the input vector, $y_{ij} \in R^p$ is the output vector and

$$[E_1, E_2] = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix}, \quad \begin{matrix} E_1 \in R^{n\times n_1} \\ E_2 \in R^{n\times n_2} \end{matrix} \quad (n = n_1 + n_2), \; A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \; \begin{matrix} E_{11}, A_{11} \in R^{n_1\times n_1} \\ E_{22}, A_{22} \in R^{n_2\times n_2} \end{matrix},$$

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \; \begin{matrix} B_1 \in R^{n_1\times m} \\ B_2 \in R^{n_2\times m} \end{matrix}, \; C = [C_1, C_2], \; \begin{matrix} C_1 \in R^{p\times n_1} \\ C_2 \in R^{p\times n_2} \end{matrix}$$

It is assumed that $\det E = 0$ and

(2) $$\det[EZ - A] = \det\begin{bmatrix} E_{11}z_1 - A_{11} & E_{12}z_2 - A_{12} \\ E_{21}z_1 - A_{21} & E_{22}z_2 - A_{22} \end{bmatrix} \neq 0$$ for some $z_1, z_2 \in \mathbf{C}$ (the field of complex numbers)

where $Z = \begin{bmatrix} I_{n_1}z_1 & 0 \\ 0 & I_{n_2}z_2 \end{bmatrix}$.

If (2) holds then

(3) $$\det[EZ - A]^{-1} = \begin{bmatrix} E_{11}z_1 - A_{11} & E_{12}z_2 - A_{12} \\ E_{21}z_1 - A_{21} & E_{22}z_2 - A_{22} \end{bmatrix}^{-1} = \sum_{i=-\mu_1}^{\infty} \sum_{j=-\mu_2}^{\infty} T_{ij} z_1^{-(i+1)} z_2^{-(j+1)}$$

where $T_{ij} \in R^{n\times n}$ are the transition matrices and (μ_1, μ_2) is the nilpotence index.

The solution to (1a) with the boundary conditions

(4) $$x^h_{0j}, j \in Z_+ \quad \text{and} \quad x^v_{i0}, i \in Z_+$$

is given by [6]

$$\begin{bmatrix} x_{ij}^h \\ x_{ij}^i \end{bmatrix} = \sum_{l=0}^{j+\mu_2-1} T_{i,j-l-1} E_1 x_{0l}^h + \sum_{k=0}^{i+\mu_1-1} T_{i-k-1,j} E_2 x_{k0}^v + \sum_{k=0}^{i+\mu_1-1} \sum_{l=0}^{j+\mu_2-1} T_{i-k-1,j-l-1} B u_{kl}$$

Consider the singular 2D system

(6) $$E\begin{bmatrix} \hat{x}_{i+1,j}^h \\ \hat{x}_{i,j+1}^v \end{bmatrix} = A\begin{bmatrix} \hat{x}_{ij}^h \\ \hat{x}_{ij}^v \end{bmatrix} + Bu_{ij} + K\left(C\begin{bmatrix} \hat{x}_{ij}^h \\ \hat{x}_{ij}^v \end{bmatrix} - y_{ij} \right)$$

where $\hat{x}_{ij}^h \in R^{n_1}$ and $\hat{x}_{ij}^v \in R^{n_2}$ are estimates of x_{ij}^h and x_{ij}^v, respectively and $K \in R^{n \times p}$.

Definition . The system (6) is called the perfect observer of (1) if and only if

(7) $$\hat{x}_{ij}^h = x_{ij}^h \text{ and } \hat{x}_{ij}^v = x_{ij}^v \text{ for } i,j \in Z_+$$

We shall establish conditions for the existence of the perfect observer (6) for (1) and derive a procedure for designing of this observer.

3. Main result.

In the sequel the following elementary operations [5, 6] will be used

Multiplication of any row (column) by a nonzero number c
Addition to any row (column) of any other row (column) multiplied by any polynomial (number)
Interchange of any two rows (columns)

Lemma. Let $E, A \in R^{n \times n}$, ($\det E = 0$) and $C \in R^{p \times n}$ be given. There exists $K = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \in R^{n \times p}$ such that

$$\det[EZ - (A + KC)] = \det\begin{bmatrix} E_{11}z_1 - (A_{11} + K_1C_1) & E_{12}z_2 - (A_{12} + K_1C_2) \\ E_{21}z_1 - (A_{21} + K_2C_1) & E_{22}z_2 - (A_{22} + K_2C_2) \end{bmatrix} = \alpha \neq 0$$

(α is a constant independent of z_1 and z_2)
if and only if

(9a) $$rank\begin{bmatrix} EZ - A \\ C \end{bmatrix} = n \text{ for all } z_1, z_2 \in \mathbf{C}$$

and

(9b) $$rank\begin{bmatrix} E \\ C \end{bmatrix} = n$$

Proof. Necessity. From the equality

$$[EZ - (A + KC)] = [\, I_n \quad K \,]\begin{bmatrix} EZ - A \\ C \end{bmatrix}$$

it follows that (8) implies (9)

Sufficiency. To simplify the notation it is assumed that $p = 1$. Using the elementary operations it is easy to show that if (9) holds then there exists nonsingular matrices $P, Q \in R^{n \times n}$ such that

$$\overline{E} = PEZQ = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1,n-1} & 0 \\ 0 & e_{22} & \cdots & e_{2,n-1} & 0 \\ \hdashline 0 & 0 & \cdots & e_{n-1,n-1} & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \quad (e_{ij} \text{ depends on } z_1 \text{ or } z_2)$$

(10)

$$\overline{A} = PAQ = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1,n-2} & a_{1,n-1} & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2,n-2} & a_{2,n-1} & a_{2n} \\ 0 & a_{23} & \cdots & a_{3,n-2} & a_{3,n-1} & a_{3n} \\ \hdashline 0 & 0 & \cdots & a_{n-1,n-2} & a_{n-1,n-1} & a_{n-1,n} \\ 0 & 0 & \cdots & 0 & a_{n,n-1} & a_{nn} \end{bmatrix}, \; a_{i+1,i} \neq 0, \; i = 1,\ldots,n-1$$

$$\overline{C} = CQ = [0 \cdots 0 \;\; 1]$$

Let (for $p=1$)

$$\overline{K}=PK=\begin{bmatrix}1-a_{1n}\\ -a_{2n}\\ \vdots\\ -a_{nn}\end{bmatrix}$$

Then

$$P[EZ-(A+KC)]Q=[\overline{E}-(\overline{A}+\overline{K}\overline{C})]=\left[\begin{array}{ccccc}e_{11}-a_{11} & e_{12}-a_{12} & \cdots & e_{1,n-1}-a_{1,n-1} & -1\\ -a_{21} & e_{22}-a_{22} & \cdots & e_{2,n-1}-a_{2,n-1} & 0\\ 0 & -a_{32} & \cdots & e_{3,n-1}-a_{3,n-1} & 0\\ \hdashline 0 & 0 & \cdots & e_{n-1,n-1}-a_{n-1,n-1} & 0\\ 0 & 0 & \cdots & -a_{n,n-1} & 0\end{array}\right]$$

and

$$\det[EZ-(A+KC)]=-\det P^{-1}\det Q^{-1}a_{21}a_{32}\cdots a_{n,n-1}\neq 0 \qquad \square$$

Example 1. For given

(12)
$$E=\left[\begin{array}{c:cc}1 & 0 & 0\\ \hdashline 0 & 1 & 0\\ 0 & 0 & 0\end{array}\right],\ A=\left[\begin{array}{c:cc}1 & 1 & 0\\ \hdashline 0 & 2 & 3\\ 1 & 0 & 0\end{array}\right],\ C=\left[\begin{array}{c:cc}1 & 0 & 1\end{array}\right]$$

find $K=\begin{bmatrix}k_1\\ k_2\\ k_3\end{bmatrix}$ such that (8) holds for $\alpha=1$.

It is easy to check that the matrices (12) satisfy the assumptions (9) since

$$rank\begin{bmatrix}EZ-A\\ C\end{bmatrix}=rank\begin{bmatrix}z_1-1 & -1 & 0\\ 0 & z_2-2 & -3\\ -1 & 0 & 0\\ 1 & 0 & 1\end{bmatrix}=3 \text{ for all finite } (z_1,z_2)\in \mathbf{C}\times\mathbf{C}$$

and $rank\begin{bmatrix}E\\ C\end{bmatrix}=rank\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\\ 1 & 0 & 1\end{bmatrix}=3$

Using the elementary operations the matrix

$$rank\begin{bmatrix}EZ-A\\ C\end{bmatrix}=rank\begin{bmatrix}z_1-1 & -1 & 0\\ 0 & z_2-2 & -3\\ -1 & 0 & 0\\ 1 & 0 & 1\end{bmatrix}$$

can be reduced to the form

$$\begin{bmatrix}z_2-2 & 3 & -3\\ -1 & z_1-1 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix} \quad \text{and} \quad P=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix},\ Q=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & -1 & 1\end{bmatrix}$$

From (11) we obtain

$$K = P^{-1}\overline{K} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$$

Theorem. Let the assumption (2) for the singular model (1) be satisfied. There exists the perfect observer (6) for (1) if and only if (9) holds.

Proof. Let us define

(14) $$\begin{bmatrix} e_{ij}^h \\ e_{ij}^v \end{bmatrix} = \begin{bmatrix} x_{ij}^h - \hat{x}_{ij}^h \\ x_{ij}^v - \hat{x}_{ij}^v \end{bmatrix} \quad \text{for } i, j \in Z_+$$

Using (14), (1) and (6) we obtain

(15) $$E\begin{bmatrix} e_{i+1,j}^h \\ e_{i,j+1}^v \end{bmatrix} = E\begin{bmatrix} x_{i+1,j}^h \\ x_{i,j+1}^v \end{bmatrix} - E\begin{bmatrix} \hat{x}_{i+1,j}^h \\ \hat{x}_{i,j+1}^v \end{bmatrix} =$$

$$= A\begin{bmatrix} x_{ij}^h \\ x_{ij}^v \end{bmatrix} + Bu_{ij} - A\begin{bmatrix} \hat{x}_{ij}^h \\ \hat{x}_{ij}^v \end{bmatrix} - Bu_{ij} + KC\begin{bmatrix} e_{ij}^h \\ e_{ij}^v \end{bmatrix} = (A+KC)\begin{bmatrix} e_{ij}^h \\ e_{ij}^v \end{bmatrix}$$

Let K be chosen so that (8) holds. Then from (3) we obtain

(16) $$[EZ-(A+KC)]^{-1} = \sum_{i=-\overline{\mu}_1}^{\infty} \sum_{j=-\overline{\mu}_2}^{\infty} \overline{T}_{ij} z_1^{-(i+1)} z_2^{-(j+1)}$$

where

$$\overline{T}_{ij} = 0 \text{ for } i \geq 0 \text{ and/or } j \geq 0$$

Hence the solution to (15) by (5) for $u_{ij} = 0$ and any boundary conditions $e_{oj}^h, j \in Z_+$, $e_{io}^v, i \in Z_+$ is equal to zero, i.e. $e_{ij}^h = 0$ and e_{ij}^v for $i, j \in Z_+$ and the condition (6) is satisfied. □

If the conditions (9) are satisfied then the perfect observer (6) for (1) can be calculated by the use of the following procedure

Procedure

Step 1. Knowing *E, A* and *C* find *K* satisfying (8) for some constant α. To compute *K* the following two approaches can be used. In the first approach *K* is chosen so that $a_{ij}(K) = 0$ for $i = 1,..., p, j = 1,..., q$ and $a_{00}(K) \neq 0$ where $a_{ij}(K)$ are coefficients of the polynomial

$$\det[EZ-(A+KC)] = \sum_{i=0}^{p} \sum_{j=0}^{q} a_{ij}(K) z_1^i z_2^j$$

The second approach is based on elementary operations and the formula (11).

Step 2. Using (6) find the perfect observer in the form

(19) $$E\begin{bmatrix} \hat{x}_{i+1,j}^h \\ \hat{x}_{i,j+1}^v \end{bmatrix} = (A+KC)\begin{bmatrix} \hat{x}_{ij}^h \\ \hat{x}_{ij}^v \end{bmatrix} + Bu_{ij} - Ky_{ij}$$

Example 2. Consider the singular model (1) with (12) and

(20) $$B = \begin{bmatrix} 2 \\ \cdots \\ 1 \\ 0 \end{bmatrix}$$

It is easy to check that the model (1) with (12) and (20) satisfies the conditions (9) (see Example 1)

Using the procedure we calculate

Step 1. From (8) for $K = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$ we obtain

(21) $$EZ-(A+KC) = \begin{vmatrix} z^1 - 1 - k^1 & -1 & -k^1 \\ -k^2 & z^2 - 2 & -k^2 - 3 \end{vmatrix} =$$

For $k_1 = 0, k_2 = -2$ and $k_3 = 0$ we obtain (8) for $\alpha = -1$. The same result we obtain using the second approach (see Example 1).

Step2. The desired perfect observer (19) has the form

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}^h_{i+1,j} \\ \hat{x}^v_{i,j+1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}^h_{ij} \\ \hat{x}^v_{ij} \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} u_{ij} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} y_{ij}$$

4. Concluding remarks.

A new concept of the perfect observer (6) for singular 2D Roesser model (1) has been proposed. Necessary and sufficient conditions for the existence of the perfect observer (6) for (1) have been established. A procedure for design of the perfect observer has been derived and illustrated by a numerical example. This new concept of perfect observer can be extended with slight modifications for singular 2D Fornasini-Marchesini type models [7,6]. An extension of these considerations for standard 2D linear systems will be considered in a separate paper.

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