OPTIMAL WAVELET-TREE DECOMPOSITION USING NOISY DATA OF IMAGES

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Abstract. Images in vision systems formed by microwave illumination are considered to be impaired by multiplicative noise. Nevertheless, there are both points corrupted by the noise and points with approximately correct values of intensity. Encoding of such images may apply a wavelet basis using any wavelet tree and thresholding of the wavelet coefficients. It's suggested the algorithm of searching for the best wavelet-tree decomposition using noisy data of images for each subband. Noisy and non-noisy data are found by applying the coefficient variance estimator.

1. Introduction

Modern vision systems use microwave illumination techniques (laser) getting images for different aims. Unfortunately, rough surfaces of details and surfaces cause random phase changes in the reflected radiation and the effect is a very noisy mottled appearance. Goodman [1] showed that such noise in general may be considered as multiplicative noise (speckle phenomena) with negative exponential probability density function (pdf) of the image intensity. The same problem appears in SAR imagery, so speckle reduction techniques can be applied here for noise smoothing. Also, it's worth to mention that infrared systems, where photoelement and respective channel coefficients of photoreceiver are floating about unity, have multiplicative noise.

Donoho [2] and Guo *at al.* [3] have investigated wavelet soft and hard thresholding procedures for restoration of noisy images. There are also some works devoted to the problem of the threshold searching. Nevertheless, the other problem of searching for the best wavelet-tree structure is still unsolved. Here we would like to note about some recognisable works. The standard Mallat's wavelet tree contains approximation branches only supposing that the most energy of signal, but not energy of noise is concentrated there. The algorithm of searching for the best wavelet-packet basis providing with the minimal total coding distortions has been suggested by H.Guo, C.S.Burrus, *at al*. [4]. For de-noising they used soft-thresholding for all of wavelet coefficients. Thus, the optimality criterion is formed for distortions during encoding of already thresholded coefficients. The other method has been created by Z.Zeng and I.Cumming [5]. They applied texture analysis with tree-structured wavelet transform. To find out the tree structure it's necessary to compare the energy *e* of a subimage with the largest energy value in the same scale, i.e.

$$e = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\left|w_{ij}\right|, \quad \text{if } e < Ce_{\max} \text{ then stop decomposition.} \qquad (1)$$

Here *C* is some constant determining the texture factor. Therefore, the algorithm is based on the energy of signal rather than the energy of noise.

We have already considered in [6] that each image both contains data which is corrupted by noise and data which is relatively "clear". It's quite possible to find out points where the noise energy is concentrated. Hence, we can analyze the noise influence within each subband to build the optimal tree structure. In the paper [6] we proposed to use the variance coefficients as indicator of texture homogeneity within the given window that allowed us to determine sets of noisy and non-noisy data.

2. Model of images

We use the multiplicative model suggested in [1] to describe the observed image y_{ij}:

$$y_{ij} = g_{ij}x_{ij}, \quad 1 \le i \le L, 1 \le j \le M, LM = A \qquad (2)$$

where g_{ij} – is the noise, x_{ij} – is the original signal. The multiplicative noise has a negative exponent distribution [1] with mean 1 and variance σ_g^2.

After logarithmic transform the multiplicative noise becomes additive:

$$z = s + \xi \qquad (3)$$

where *z* denotes *ln*(*y*), *s* = *ln*(*x*) and ξ = *ln*(*g*).

3. Determining sets of noisy and non-noisy data

At first we determine three sets of points: 1) where the original dominates noise; 2) homogeneous region where an averaging filter can restore the original; and 3) where the situation is not so clear, that is, smoothing the noise is needed.

To do it we use the ratio of coefficients of variance as indicator of homogeneity within the current window:

$$C_z = \sigma_z^2 / \mu_z^2 , \qquad C_\xi = \sigma_\xi^2 / \mu_\xi^2 , \tag{4}$$

where $\mu_z = \bar{z}$, $\mu_\xi = \bar{\xi}$ - mean of signal and noise respectively calculated within two windows, with a small size window for the noise; σ_z^2, σ_ξ^2 stand for variances. Then we can determine:

$$\tilde{s} = \begin{cases} \bar{z}, & C_z \le C_\xi \\ z, & C_\xi < C_z < C_{\max} \\ \tilde{z}, & C_z \ge C_{\max} \end{cases} \tag{5}$$

where $C_{\max}$= max{C_ξ}.

Then the three sets of points (4) are being combined into two sets, the noisy data set $\tilde{s}_N$ from z containing A_N points, the non-noisy data set $\tilde{s}_S$ is the unity of the rest A_S points; $A_N+A_S=A$.

4. Description of the algorithm

The main aim of the algorithm is to find the optimal wavelet tree structure in the sense of some criterion. This criterion should be formulated as maximum energy value of the noise to be eliminated during analysis of noisy points within each subband.

For the orthogonal wavelet transformation with given threshold we can check the influence of noisy component for each subband. Indeed, if the transformation is orthogonal then we can restore by inverse wavelet transform that part of image which corresponds to the given subband. The coefficients of the rest of the subbands aren't taken into consideration.

The algorithm consists of several steps.

1. To determine the sets of noisy and non-noisy data (see p.2) and to calculate the threshold value using noisy data: $t = \sigma_N \sqrt{\log(A_N)}$

2. To perform wavelet-packet decomposition of the given image for L levels.

3. A hard thresholding method allows to us to eliminate with threshold t those of coefficients which represent the noise:

$$\hat{W} = \begin{cases} W, & if \quad |W| > t \\ 0, & if \quad |W| \le t \end{cases} \tag{6}$$

4. Using estimated wavelet coefficients $\hat{W}$ only for the given subband we can recover the de-noised image for corresponding frequency. On the other hand, we can restore the noise component if we perform an inverse transform of:

$$\breve{W} = \begin{cases} W, & if \quad |W| \le t \\ 0, & if \quad |W| > t \end{cases} \tag{7}$$

5. For points corresponding noisy data A_N to perform analysis from the bottom of the decomposition:

$$\frac{1}{2}\sum_{k=1}^{4} e_{k,l} < e_{k,l-1}, \text{where } e_{k,l} = \frac{1}{N_l}\sum_i \sum_j \left|\tilde{\xi}_{ij}^{(l)}\right|, \tag{8}$$

here l - the level of decomposition, l=1,..,L, k - number of one among four subbands within the subband of the high level, N_l – the quantity of noisy points.

5. The results of modelling

This algorithm was tested on a few images including real laser holograms, infrared and SAR images. All experiments showed the de-noising capability of the suggested algorithm. The images used here are taken from the MATLAB library to demonstrate the difference between original and restored images. We tested the algorithm by mathematical modeling of the real situation.

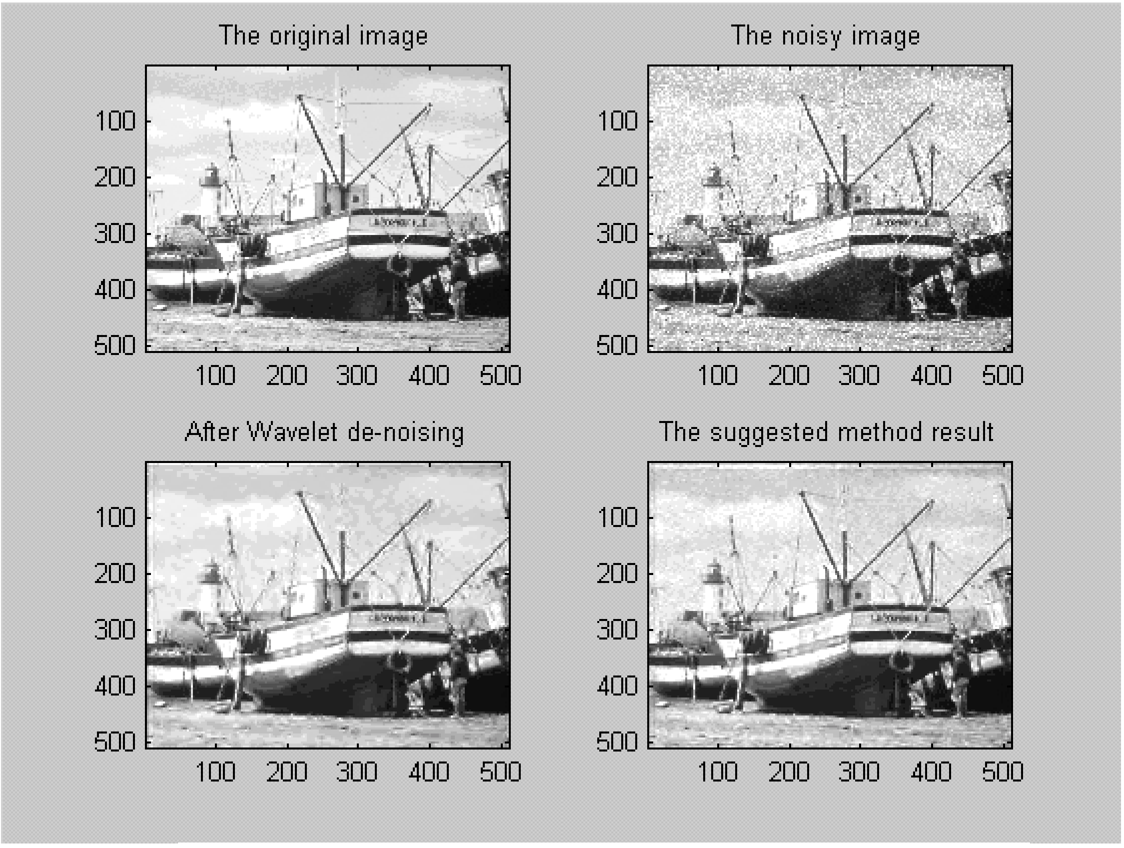

Fig.1-4. The original image and the results of modeling

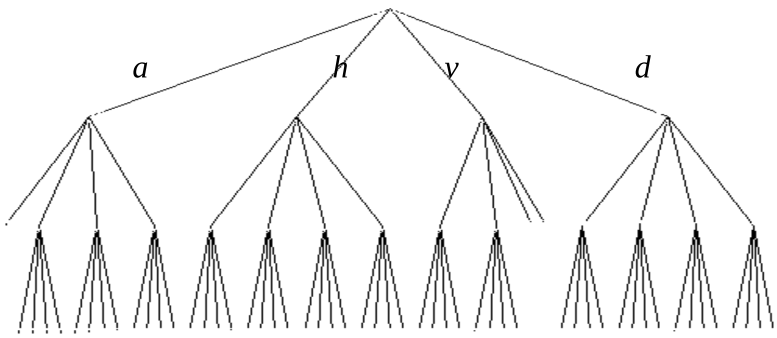

Fig.5. The wavelet tree structure of 3-level decomposition; *a* – approximation, *h* – horizontal, *v* – vertical, and *d* – diagonal

The original image "Boat" (fig.1) has been distorted with multiplicative noise gained by a random number generator. The noise had the negative exponent pdf with mean 1 and variance σ_g^2=0,33. The minimal signal/noise ratio achieved was about 3. Before wavelet transformation this image has been undergone by logarithmic transform. The window size 3×3 was applied for the noise variance estimator.

We used the three-level wavelet decomposition not keeping the approximation coefficients. The Daubechies 4 wavelet base was used for the wavelet filter.

The results of modeling are shown on figs.2-5. The fig.5 shows the wavelet tree structure. The mean square error (MSE) of distortion and restoration are placed in

Table. All the MSE values were calculated as the difference between the noisy image, restored images and the original image. In comparing we used the standard Mallat's wavelet tree and the algorithm [5] with quadtree searching procedure (1). The results from Table show that our algorithm allows us to get a better MSE even comparing with the MSE of the famous methods for three level of wavelet decomposition.

Visually, the algorithm provides better noise smoothing while maintaining edge sharpeness.

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Table of results

Type of MSE	The full error	Mallat	Quadtree	The suggested method
Noisy data	18.02	12.05	11.78	11.01
All of data	21.83	11.88	11.38	10.03