A NEW APPROACH IN IMAGE CLASSIFICATION

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ABSTRACT - A simply implemented algorithm for analysis and recognition of images is presented. The suggested method is based on Toeplitz matrices and their determinants. The algorithm classifies the image characteristic points into a feature vector whose elements form the essential entry data to Toeplitz matrices. The minimal eigenvalues of these matrices are calculated. They form a monotonically nonincreasing series whose graph and sequence limit are of characteristic values. These values simply correspond to the original image shape.

1. Introduction

One of the basic steps in computer image analysis for recognition is the feature extraction and their simple way of description for classification and hence description. In this work a trial for a new approach of image classification is given. The method is based on calculating the minimal eigenvalues for a special class of Toeplitz matrices [1,2]. These eigenvalues form the elements of the feature vector necessary for classification and comparison with the available data. Since Toeplitz matrix elements carry the specific characteristics of the image [3], then their corresponding eigenvalues give a really good virtual description of the original image. This was experienced on different types and classes of images from written letters [4] to words [5] or signatures [6] and spoken letters [7]. Here the main ideas and calculation procedure are introduced with an example on script classification to show the efficient sides of the author's approach in comparison to the other known to him methods of classification and description.

2. Mathematical Background

Assume that the feature *n*-points extracted from an image on the *x-y* plane form the coefficients of the following rational function of the complex variable $s = \sigma + j\omega$:

$$f(s) = \frac{x_0 + x_1 s + x_2 s^2 + \ldots + x_n s^n}{y_0 + y_1 s + y_2 s^2 + \ldots + y_n s^n} \tag{1}$$

Making the transformation $f(s) \to f(z)$, by putting $s = \frac{1-z}{1+z}$, we get another class of rational functions, regular in the unit circle $|z| < 1$ and defined as:

$$f(z) = \frac{u_0 + u_1 z + u_2 z^2 + \ldots + u_n z^n}{v_0 + v_1 z + v_2 z^2 + \ldots + v_n z^n} \tag{2}$$

A power series (Taylor series) is formed from the coefficients of the numerator and the denominator of *Eq*.(2). It has the following form:

$$f(z) = \alpha_0 + \alpha_1 z + \alpha_2 z^2 + \ldots + \alpha_n z^n + \ldots \tag{3}$$

where

$$\alpha_0 = \frac{u_0}{v_0}, \quad \alpha_i = (y_0)^{-i-1} \begin{vmatrix} u_i & v_1 & v_2 & \ldots & v_i \\ u_{i-1} & v_0 & v_1 & \ldots & v_{i-1} \\ \ldots & 0 & v_0 & \ldots & \ldots \\ \ldots & \ldots & 0 & v_0 & v_1 \\ u_0 & 0 & 0 & 0 & v_0 \end{vmatrix}$$

for $i = 1, \ldots, n$.

Then, form Toeplitz matrices from these coefficients. Their determinants are given as follows

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$$D_0 = c_0,\ D_i = \begin{bmatrix} c_0 & c_1 & c_2 & \dots & c_n \\ c_1 & c_0 & c_1 & \dots & c_{n-1} \\ \dots & \dots & \dots & \dots & \dots \\ c_n & c_{n-1} & c_{n-2} & \dots & c_0 \end{bmatrix} \qquad (4)$$

assuming that $\alpha_0 = c_0,\ \alpha_n = c_n,\ c_{-n} = \bar{c}_n$, n=1,2,3, ...

Now, the minimal eigenvalues $\lambda_{\min}^{(n)}$ of these determinants are calculated for $i = 1,\dots,n$. The λ's form a monotonically nonincreasing sequence whose limit approaches a specific value, theoretically tends to the minimal value of the real part, denoted by $\Re(f)$, of the function $f(s)$ of $Eq.(1)$ at $s = j\omega$ 8]. This plays essential role in Circuit Theory when considering the class of positive real rational functions and their applications in the digital filters and their realization [1]. These details are beyond the topics of this paper. However, the steering theory in Toeplitz forms and their determinants lies in the behavior of analytic functions and their applications. To explain that, consider the following matrices:

$$M = \begin{bmatrix} \alpha_0 & \alpha_1 & \alpha_2 & \dots \\ 0 & \alpha_0 & \alpha_1 & \dots \\ 0 & 0 & \alpha_0 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix} \qquad M^* = \overline{M'} = \begin{bmatrix} \bar{\alpha}_0 & 0 & 0 & \dots \\ \bar{\alpha}_1 & \bar{\alpha}_0 & 0 & \dots \\ \bar{\alpha}_2 & \bar{\alpha}_1 & \bar{\alpha}_0 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

$$[H] = \frac{1}{2}\left[M + \overline{M'}\right]$$

$$= \frac{1}{2}\begin{bmatrix} 2\Re\alpha_0 & \alpha_1 & \alpha_2 & \dots \\ \bar{\alpha}_1 & 2\Re\alpha_0 & \alpha_1 & \dots \\ \bar{\alpha}_2 & \bar{\alpha}_1 & 2\Re\alpha_0 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

Assuming $\alpha_0 = c_0,\ \alpha_n = 2c_n,\ c_{-n} = \bar{c}_n$, n=1,2,... .

Then,

$$[H] = \begin{bmatrix} c_0 & c_1 & c_2 & \dots \\ c_{-1} & c_0 & c_1 & \dots \\ c_{-2} & c_{-1} & c_0 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix} \qquad (5)$$

where [*H*] is the infinite Hermitian matrix and is of Toeplitz type. Now, the following holds:

THEOREM [2]

Let $f(z) = c_0 + 2\sum_{n=1}^{\infty} c_n z^n$, c_0 real, $c_{-n} = \bar{c}_n$, be regular in the unit circle $|z|<1$. Denote by $\lambda^{(n)}$ the lowest eigenvalue of the Toeplitz matrix:

$$\begin{bmatrix} c_0 & c_1 & c_2 & \dots & c_n \\ c_{-1} & c_0 & c_1 & \dots & c_{n-1} \\ \dots & \dots & \dots & \dots & \dots \\ c_{-n} & c_{-n+1} & c_{-n+2} & \dots & c_0 \end{bmatrix},$$

which is the *n*th order of the infinite Hermitian matrix defined by $Eq.(5)$. The quantities $\lambda^{(n)}$ form a nonincreasing sequence and

$$\lim_{n\to\infty} \lambda^{(n)} = \min \Re f(z)$$

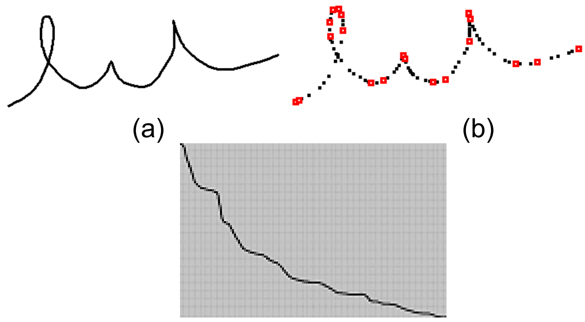
Therefore, this theorem gives an effective tool for feature classification. Experiments [1,3,4,5] have shown that the monotonically nonincreasing sequence:

$$\lambda_0 \geq \dots \geq \lambda_i \geq \dots \geq \lambda_n,\ i = 1,\dots,n-1 \qquad (6)$$

shows a characteristic plot for a given image differing from others. For a definite language scripts [4], for example, each letter belongs to a certain class differing completely from the others.

3. EXAMPLES AND CONCLUSIONS

In Fig.1a an image is plotted in the *x-y* plane. Fig1b shows its characteristic points, which form the essential feature vector components. Following the theory above, Fig.1c is obtained as a sketch of the minimal eigenvalues sequence.

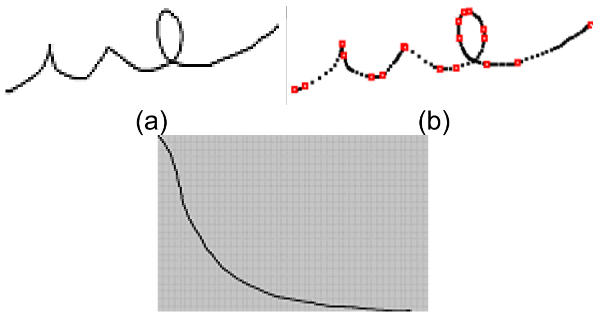


(a) (b)

(c)

Fig.1 *Extraction of feature vector and eigenvalues:* (a) *A random image.* (b) *Its characteristic points.* (c) *Minimal Eigenvalues sketch.*

Now consider another image with almost the same characteristic points. In a similar way, evaluate the minimal eigenvalues (Fig.2). Although the images generally have similar shape, they furnish different eigenvalue sketches (compare Fig.1c with Fig.2c).



(a) (b)

(c)

Fig.2 *Another image* (a)*, its features* (b)*, and the minimal eigenvalue sequence sketch* (c).

The author is working on algorithm application to different classes of images. The original approach has also been modified from feature point extract point of view. Future work will cover these and other practical sides of the main criterion.

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