POSITIVE-REALNESS IN REPETITIVE AND ITERATIVE LEARNING CONTROL - AN EXTENDED ABSTRACT

J. Hätönen[*,**] D. H. Owens[*], K. Feng[*], V. Hatzikos[*]

[*]Department of Automatic Control and Systems Engineering,
University of Sheffield, Mappin Street, Sheffield S1 3JD, UK
[**]Systems Engineering Laboratory, University of Oulu, P.O. BOX 4300,
FIN-90014 University of Oulu, Finland

1. Introduction

Iterative Learning Control (ILC) and Repetitive Control (RC) are rather new techniques to control systems that work in a repetitive mode. These control systems form a subclass of 2-D systems, because the system has two independent axes, the time axis and the repetition axis.

Recently, Positive Real (PR) systems have aroused a lot of interest both in RC and ILC research (Galkowski et al, 2002) because they usually give good convergence properties with reasonably simple algorithms. In this paper it is shown, however, that PR systems are not necessarily useful in continuous-time RC, because the (necessary) digital implementation of a continuous-time RC algorithm will destroy the stability properties of the algorithm. In the discrete-time ILC case, on the other hand, it is shown that a simple adaptive ILC algorithm will result in convergent learning, if the plant $zG(z)$ is PR, demonstrating the possible importance of positive real systems in ILC. Simulations highlight the theoretical findings in this paper.

2. Positive-Realness in Repetitive Control – sampling destroys stablity

As a starting point in continuous-time RC it is assumed that a mathematical model

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0 \\ y(t) = Cx(t) \end{cases} \tag{2.1}$$

of the plant in question exists where $t \in [0, \infty)$, and A,B and C are matrices of appropriate dimensions. Furthermore, a reference signal $r(t)$ is given, and it is known that $r(t+T) = r(t)$ for a known T (in other words the actual shape of $r(t)$ is not necessarily known). There exits a large variety of important control applications that fit into the RC framework, examples being robotics, rotating mechanisms and rolling processes.

The design objective is to find a feedback controller that makes the system (1) to track the reference signal as accurately as possible under the assumption that the reference signal is T-periodic. As was shown in (Francis and Wonham, 1975), a necessary condition for zero convergence in the limit is that the control law

$$[Mu](t) = [Ne](t) \tag{2.2}$$

where M and N are suitable operators, has to have an internal model of the reference signal in the M operator. Because $r(t)$ is T-periodic, its internal model is $1\text{-}\sigma_T$, where $[\sigma_T\, v](t)=v(t-T)$ for a arbitrary time function $v(t)$. Hence for example in (Yamamoto, 1993) it was suggested that one possible (and obviously computationally simple) RC algorithm is

$$u(t) = u(t-T) + e(t) \tag{2.3}$$

This algorithm has been analysed by several authors (see for example (Yamamoto, 1993), (Arimoto and Naniwa, 2000) and (Owens *et al*., 2001)), and it turns out that a sufficient condition for convergence is that the system is positive real (PR). In this paper it is shown that in the SISO-case the algorithm (2.3) converges in the limit to a feedback controller $u(t)=Ke(t)$ where $K \to \infty$ and PR systems are the only dynamical systems that can cope with infinite feedback gain, resulting in a nice interpretation for the earlier results. The underlying problem, however, is that due the delay element in the algorithm (2.3), this algorithm can be never implemented with analogue components. In this paper it is shown, however, that a plant (A,B,C) that is sampled with zero-order hold is never PR as a discrete-time system and the discrete-time implementation of (2.3) can easily result in instability even if the underlying continuous-time process model (2.1) is positive-real.

3. Positive-Realness in Iterative Learning Control

In discrete-time Iterative Learning Control the starting point is a system model

$$\begin{cases} x(t+1) = Ax(t) + Bu(t), \quad x(0) = x_0 \\ y(t) = Cx(t) \end{cases} \tag{3.1}$$

where $t \in [0, T]$. Furthermore, a reference signal $r(t)$ is given, and the control objective is to make the system (3.1) to follow this reference signal as accurately as possible. The special feature of the ILC problem is that after the system has reached the final time point $t = T$, the state of the system is reset to initial condition x_0, and after the resetting the system is supposed to track the same reference signal again. Practical applications that fit into this framework can be found for example in robotics and chemical batch processing.

The fact that the system works in a repetitive mode opens up possibilities for using information from previous repetitions (or “trials” or “iterations”) to come up with a new input function that gives better tracking performance. One of the first algorithms was

$$u_{k+1}(t) = u_k(t) + \gamma e_k(t+1) \tag{3.2}$$

where γ is a “learning gain” and k is the repetition number. It can be shown that the tracking error will converge to zero if $r(0)=Cx_0$ and $|1- \gamma CB|<1$ (Moore, 1993). However, the algorithm suffers frequently from bad transient behaviour (i.e. the norm of the tracking error can be extremely large during earlier repetitions, see (Longman, 2000)), and more advanced algorithms are needed. One possible approach from (Owens and Feng, 2002) is to use an adaptive algorithm

$$u_{k+1}(t) = u_k(t) + \gamma_{k+1} e_k(t+1) \tag{3.3}$$

where γ_{k+1} is the solution of the optimisation problem

$$\min_{\gamma_{k+1}} \left\{ \|e_{k+1}\|^2 + w\gamma_{k+1}^2 \right\} \tag{3.3}$$

where w is a weighting parameter and the norm $\|\cdot\|$ is the Euclidian norm. A simple interlacing result shows that $\|e_{k+1}\| \le \|e_k\|$ resulting in *monotonic convergence* and the optimal γ_{k+1} is given by the equation

$$\gamma_{k+1} = \frac{e_k^T G_e e_k}{w + e_k^T G_e^T G_e e_k} \tag{3.4}$$

where

$$G_e = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ CA^2B & CAB & \ddots & \vdots \\ CA^{T-1}B & CA^{T-2}B & \cdots & CB \end{bmatrix} \tag{3.5}$$

Furthermore, it will be shown in the paper that a sufficient condition for zero convergence is that G_e is a positive-definite matrix and sufficient for G_e to be positive-definite is that the transfer function $zG(z)$ (where $G(z) = C(zI-A)^{-1}B$) is a positive real system in the context of discrete time systems (for the definition n of a PR system in the discrete-time case see (Desoer and Vidyasagar, 1975)).

4. Conclusions

In this paper the importance of positive-realness in RC and ILC has been discussed. In RC it turns out that positive-realness is required in the continuous-time case for convergence with the control law (2.3). However, sampling will destroy positive-realness, and in practise the control (2.3) will not work, even a considerable amount of research work has been published on this algorithm (see for example (Arimoto and Naniwa, 2000) and references therein).

In ILC, on the other hand, it seems that positive-realness and adaptive Iterative Learning Control are closely connected. This is due the fact that the simple adaptive algorithm (3.3) will converge, if modified system matrix (3.5) is positive-definite. Furthermore, if the trial length approaches infinity, this condition is equivalent that $zG(z)$ is PR (and not $G(z)$), broadening the applicability of this algorithm.

References

1. Anderson, B.D.O and S. Vonpanithred (1973). *Network Analysis and synthesis: A modern system theory approach.* Prentice Hall, Englewood Cliffs, NJ.
2. Arimoto, S. and T. Naniwa (2000). Equivalence relations between learnability, output-passivity and strict positive realness. *International Journal of Control* **73**(10), 824-831.
3. Francis, B.A. and W.M. Wonham (1975). The internal model principle for linear multivariable regulators. Appl. Math. Opt. pp.107-194.
4. Longman, R.W. (2000). Iterative Learning Control and Repetitive Control for engineering practise. *International Journal of Control* **73**(10), 930-954
5. Moore K. (1993). *Iterative Learning Control for Deterministic Systems*. Springer-Verlag.
6. Ogata, K. (1973). *Discrete-time Control Systems.* Prentice-Hall, Englewood Cliffs, NJ.
7. Owens, D.H and K. Feng (2002). *Parameter Optimisation in Iterative Learning Control*. Research Report 822. Department of Automatic Control and Systems Engineering, The University of Sheffield.
8. Owens, D.H., L.M. Li and S.P. Banks (2001). MIMO-multiperiodic repetitive control systems: a Liapunov analysis. In: Proceedings of the IFAC Workshop on periodic systems and control.
9. Yamamoto, Y. (1993). Essays on Control: Perspectives in theory and its applications. Chap. Learning Control and Related Problems in Infinite-Dimensional Systems, pp. 191-222. Birkhauser.
10. Galkowski K., Paszke W., Sulikowski B., Xu S., Rogers E., Lam J., Owens D.H., Positive realness and the analysis of a class of 2D linear systems, *IFAC 15th World Congress* 2002, Barcelona, Spain, 21-26 July, 2002.