SUCCESSIVE STABILIZATION OF HIGH DIMENSIONAL REPETITIVE PROCESSES

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Abstract. In this paper we report further significant progress on the development of a mature systems theory for discrete linear repetitive processes which are a distinct class of 2D discrete linear systems of both systems theoretic and applications interest. Here we first propose an extension to the basic state space model of these processes to include coupling terms previously neglected but which could arise in applications. Then we develop some significant first results on the analysis and control of examples represented by this new model structure.

Key Words Repetitive dynamics, controllability, stability, 1D equivalent model, stabilization.

1 Introduction

The essential unique characteristic of a repetitive, or multipass, process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the next pass profile. This, in turn, leads to the unique control problem for these processes in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass to pass direction. Physical examples of repetitive processes include long-wall coal cutting and metal rolling operations [4]. Also in recent years applications have arisen where adopting a repetitive process setting for analysis has distinct advantages over alternatives. Examples of these so-called algorithmic applications of repetitive processes include classes of iterative learning control schemes [1] and iterative algorithms for solving nonlinear dynamic optimal control problems based on the maximum principle [2].

Attempts to control these processes using standard (or 1D) systems theory/algorithms fail (except in a few very restrictive special cases) precisely because such an approach ignores their inherent 2D systems structure, i.e. information propagation occurs from pass to pass and along a given pass together with resetting of the initial conditions before the start of each new pass.

In common with a large range of other areas in systems theory, recent years has seen the emergence of Linear Matrix Inequality (LMI) based techniques in the the analysis of very important sub-classes of both differential and discrete linear repetitive processes. This has led to considerable success, especially in areas such as the structure and design of control laws and stability in the presence physically relevant types of uncertainty in the matrices of the defining state space model which have proved difficult to advance using other analysis tools.

It is important to note, however, that this progress has often required the preliminary step of building a standard (termed 1D) linear system equivalent model of the underlying process dynamics resulting in very large dimensioned LMIs to be solved. Also the model structure studied neglects a term which could be very important in some applications areas. In this paper, we first propose a model structure to capture this missing term and then develop significant new results on the control related analysis of processes described by this new model. These results consist of those which are the (non-trivial) extension of those already in existence for other models of discrete linear repetitive processes (e.g. pass controllability) and others on so-called successive stabilization which are completely new. We begin in the next section by defining the new model structure.

2 Background

The state space model of a so-called extended discrete linear repetitive process is described by the following state space model over $p = 0,1,...,\alpha - 1$.

$$x_{k+1}(p+1) = Ax_{k+1}(p) + B_0 y_k(p) + Bu_{k+1}(p), \quad (1)$$

$$y_{k+1}(p) = Cx_{k+1}(p) + D_0 y_k(p) + Du_{k+1}(p), \quad (2)$$

Here on pass *k*, $x_k(p) \in \Re^n$ is the state vector, $y_k(p) \in \Re^m$ is the pass profile vector, and $u_k(p) \in \Re^l$ is the vector of control inputs.

In order to complete the process description, it is necessary to specify the boundary conditions, i.e. the pass state initial vector sequence and the initial pass profile. This is a critical task since it is known that the structure of these initial, or boundary, conditions alone can cause instability for the discrete linear repetitive process state space models considered todate. Here these are assumed to be of the form

$x_k(0)=d_k(0) \in \Re^n$, k = 1,2, …, $y_0(p)=f_0(p) \in \Re^m$ $p = 1,2,...,\alpha-1$, where *dk* is a vector with known constant entries and *y(p)* is a vector whose entries are known functions of $p = 0,1,...,\alpha-1$.

Motivation for considering processes of the form (1)-(2) arises from applications where the current pass profile at any point along the pass is a function of more than one point on the previous pass.

In the discrete linear repetitive process state space model, first proposed in [3], it was assumed that the current pass and state profile vector was only directly influenced by the pass profile vector at the same point on the previous pass. Here we study the case when in fact all points along the previous pass directly influence the state and pass profile vectors at any point on the current pass.

2.1 1D equivalent model

For considered class of processes (1)-(2) direct 2D approach is very hard to apply. Hence, it is comfortably to build a 1D equivalent model. In such a model dynamics along the pass is hidden. This is possible to do because of the finite duration of the pass (α points). In result, the model with dynamics from pass to pass only is obtained and due to some restrictions it can be considered as an ordinal 1D model. To obtain 1D model the following two steps are required to be done. The first step is to introduce the following substitutions into (1)-(2) $l = k+1$, $y_{l-1} = v_l$ to obtain the 2D linear systems 'Roesser–like' model. Next, introduce so-called global state, input and pass profile vectors. Combining the results (state and output) for every point on the pass provides mentioned 1D equivalent model

$$X(l) = \Gamma V(l) + \Sigma U(l) + \Psi_0 x_l(0) \qquad (3)$$

$$V(l) = \Phi V(l) + \Delta U(l) + \Theta_0 x_l(0) \qquad (4)$$

where $\Phi, \Delta, \Theta_0, \Gamma, \Sigma$ and Ψ_0 are appropriately dimensioned 1D model matrices. The structures of 1D matrices are proper and regard to 2D model. Note that, (3) becomes a state observer and (4) plays a role of state equation here now.

2.2 Analysis and synthesis of 1D equivalent model

The stability investigation and controller design 2D approach is a very difficult to obtain for LRP described by (1)-(2). However, using the 1D equivalent model it has been proven (see [9]) that the process described by 2D equations (1)-(2) is asymptotic stable iff $\rho(\Phi) < 1$, where $\rho(\cdot)$ denotes the spectral radius of $(\cdot)$ value. Moreover, using the Lyapunov approach, note that equation (4) is asymptotic stable iff the following inequality is satisfied for some *P > 0*

$$\Phi^T P \Phi - P < 0 \qquad (5)$$

where *<0 (>0)* denotes the positive (negative) definiteness of the matrix. If the model has been found to be unstable, it is possible to design the stabilising controller of the form

$$U(l) = KV(l), \qquad (6)$$

where *K* is an appropriately dimensioned matrix to be designed. Also under this law the resulting closed loop process is asymptotically stable if, and only if, $\rho(\tilde{\Phi}) < 1$, where

$$\tilde{\Phi} = \Phi + \Delta K, \qquad (7)$$

To select *K*, here we can use a standard LMI setting and, in particular, the following result,

Theorem 1. Suppose that a discrete linear repetitive process of the form (5)-(6) is subject to a control law of the form (6). Then the resulting closed loop process is asymptotically stable if, and only if, $\exists$ *matrices* $P = P^T > 0$ *and G, L of appropriate dimensions, such that the following LMI holds*

$$\begin{bmatrix} -P & \Phi G + \Delta L \\ G^T \Phi^T + L^T \Delta^T & P - G - G^T \end{bmatrix} < 0 \qquad (8)$$

and if this is the case then a stabilizing control law matrix is given by $K = LG^{-1}$.

3 Successive stabilization of the 1D equivalent model

The analysis in the previous section has produced a systematic method of ensuring asymptotic stability of a discrete linear repetitive process of the form considered here using the 1D equivalent model. A consequence of this, however, is that the inherent 2D linear systems structure of these processes has been subsumed into the 1D model. In some cases of interest, it will be important to retain the 2D systems structure for other purposes. To proceed, suppose that the feedback control law matrix K is assumed to be of the form appropriate to retain the repetitive process structure after stabilization. It is achieved by assuming that *K* consists of the set of the same rows in the form $K_i = [K_1\ K_2\ ...\ ,K_{\alpha-1}]$, where $i=1,2,\ldots,\alpha$ denotes matrix row number. A basic problem in the design of control schemes for discrete linear repetitive processes is that exploiting the 1D equivalent model enables much theoretical progress the resulting matrices to be manipulated may well be of unacceptably high dimensions. In the remainder of this section we develop the concept of so-called successive stabilization as a possible answer to this difficulty. The basic idea is that we first make the process asymptotically stable over a short pass length and then subsequently augment this design. To use this approach, it is essential to only use control action which preserves the original repetitive process state space model structure. The main idea here is to implement the iterative algorithm that provides the sequence of K^j, (j denotes the iteration number) which starts from Φ of little dimensions and then successively increases them to the full size of Φ if it is required. The numerical tests prove, that in some cases proposed method makes it possible to stabilize the system which cannot be stabilized using ”the whole Φ at once” stabilization approach.

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