LIBRARIES OF ANISOTROPIC NON-DYADIC HAAR BASES

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Abstract. A scheme for building libraries of anisotropic non-dyadic two-dimensional Haar bases is presented. Generalized Fibonacci *p*-trees are used to derive the space division structure of the bases. Application of the algorithm to binary images is investigated and performance versus standard dyadic Haar bases library is compared .

1. Introduction

Finding proper basis for signal decomposition is crucial in signal processing applications where extraction of important information from large amount of raw data is a primary task. A natural solution is to build a library of bases and choose the best one for a given signal according to some cost measure [4]. In this case a problem arises with constructing an efficient search algorithm for finding the best basis. A most promising scheme is provided by wavelet packets which give a fast *O(NlogN)* time search algorithm organizing the library of bases into a dyadic tree structure. As Wickerhauser points out in [1], wavelet packets are very appropriate for 1-D signals, however for 2-D signals, i.e. images they are confined to a limited quad-tree isotropic subset of the whole library of possible bases. A recent resolve to this search problem is reported by Bennett in [2]. For 2-D or higher dimension signals he describes an *NlogN* algorithm for finding best basis within a full library of anisotropic Haar, Walsh or Cosine bases. The algorithm is built upon a graph representation of the library and an efficient iterative bottom-up search procedure minimizing an entropy-based cost function. Although it finds the best basis from the specified libraries, the optimality of the scheme is limited by the dyadic nature of the bases.

In this article we propose an algorithm for embedding a non-dyadic tree-structured Haar transform into the best-basis search method described in [2]. We use generalized Fibonacci *p*-trees to derive the time (space) division structure of the transform as developed in [3]. The new scheme is tested for feasibility in processing of binary images.

2. Generalized Fibonacci *p*-trees.

A generalized Fibonacci *p*-sequence [5] is derived from the following formula:

$$\phi_p(i) = \begin{cases} 0, & i < 0 \\ 1, & i = 0 \\ \phi_p(i-1) + \phi_p(i-p-1), & i > 0 \end{cases} \quad (1)$$

where $p \in \mathrm{N}$. Table 1 gives some examples of generalized Fibonacci *p*-sequences.

Table 1. Generalized Fibonacci p-sequences

$\phi_p(i)$	$i=0$	1	2	3	4	5	6	7	8	9
$p=0$	1	2	4	8	16	32	64	128	256	512
1	1	1	2	3	5	8	13	21	34	55
2	1	1	1	2	3	4	6	9	13	19
3	1	1	1	1	2	3	4	5	7	10

Fibonacci *p*-representation of a natural number *B* can be acquired by:

$$\mathrm{B} = \sum_{i=p}^{n-1} a_i \phi_p(i), \qquad a = (a_{n-1}, \dots, a_p)_p \text{ - Fibonacci } p\text{-code for } B \quad (2)$$

An example Fibonacci 1-code is given in Table 2 and a corresponding generalized Fibonacci 1-tree is shown on Fig.1.

Table 2. Fibonacci 1-code

	a
	00000
	00001
	00010
	00100
	00101
	01000
	01001
	01010
	10000
	10001
0	10010

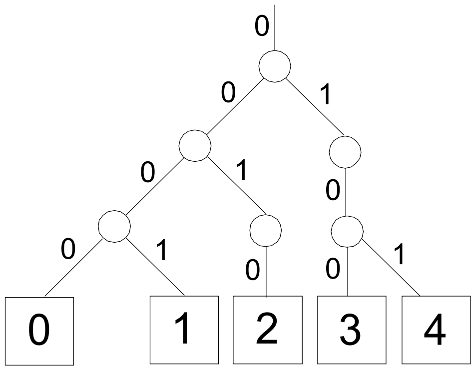

Figure 1. Generalized Fibonacci 1-tree for natural numbers 0 – 4

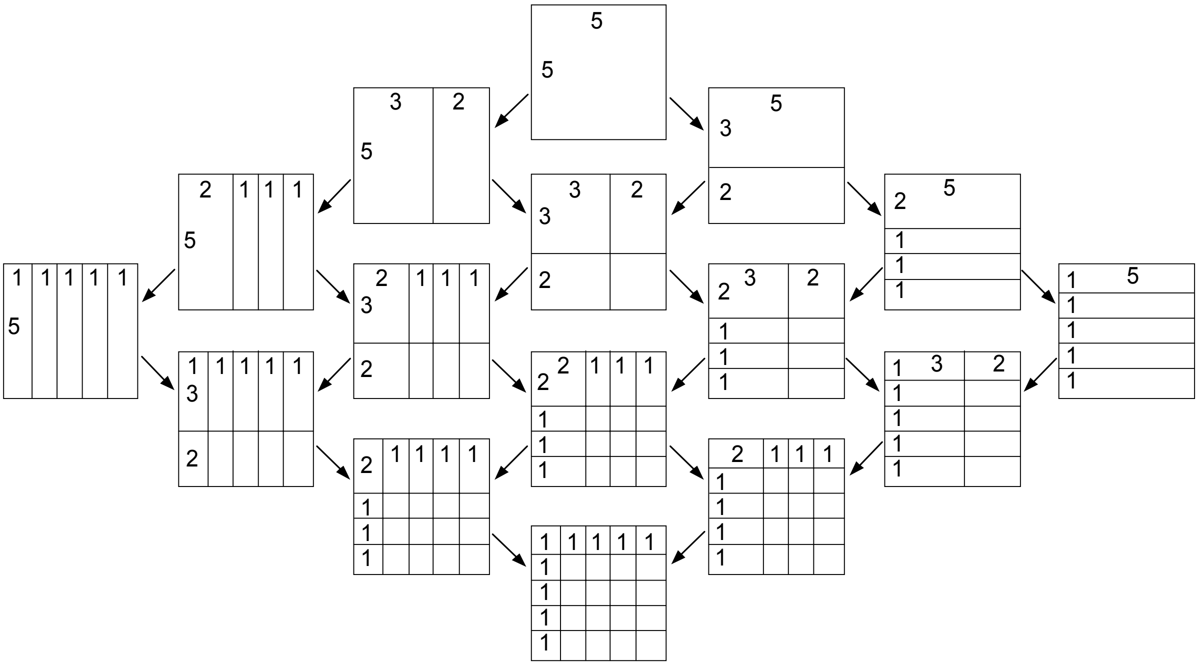

Figure 2. Library of tree-structured anisotropic generalized Fibonacci 1-tree Haar bases for an 5x5 image. The topmost member is the original image. From top to bottom time (space) resolution is reduced and frequency (scale) resolution is increased.

3. Libraries of anisotropic non-dyadic tree-structured Haar bases.

Generalized Fibonacci *p*-trees can be used for constructing non-dyadic tree-structured Haar transforms as described in [3]. Extending this notion from 1-D to 2-D and employing the library construction scheme from [2] leads to the realization of a new library of anisotropic non-dyadic tree-structured Haar bases. The fast best-basis search algorithm from [2] is readily applicable to this new scheme. An example library for an 5x5 image and $p = 1$ is presented in Fig.2.

This scheme is tested on 8x8 images of English letters and digits with $p = 1$ and compared against the standard dyadic one. Entropy is used as an additive cost measure and means for comparison. Table 3 summarizes some of the obtained results.

Table 3. Entropies for different pictures and different bases expansions

Character	'0'	'1'	'2'	'3'	'4'	'5'	'6'	'7'	'8'
Non-dyadic	**2.2523**	**1.1424**	**2.3122**	**2.3547**	**1.9695**	2.3291	2.2902	2.3822	**2.0999**
Dyadic	2.2668	1.3842	2.5228	2.3843	2.1174	**2.2822**	**2.1434**	**2.3373**	2.1562

Character	‘9’	‘a’	‘b’	‘c’	‘d’	‘e’	‘f’	‘g’	‘h’
Non-dyadic	**2.1577**	**1.7190**	2.0161	2.2777	**2.0548**	1.8526	2.2425	2.1819	1.5689
Dyadic	2.1645	1.7261	**1.8906**	**2.1489**	2.2831	**1.7348**	**1.8336**	**2.0482**	**1.4589**

4. Conclusion

The comparative results show that there are cases in which the non-dyadic scheme gives lower entropy of the transform coefficients than the dyadic one. This rises the idea that the optimal solution is to use both schemes interchangeably. A promising direction for future research in this area is the development of a generalized tree-structured Haar bases library incorporating Haar transforms with different p into one homogenous scheme.

A possible application of this algorithm can be processing of binary images such as binarized characters in optical character recognition systems.

References

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