ADAPTIVE CHANNEL EQUALIZATION IN NON-GAUSSIAN NOISE ENVIRONMENTS

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Abstract. The algorithm presented in this paper belongs to the class of Order Statistic LMS adaptive algorithms (OSLMS), that uses a smoothed gradient to update the filter coefficients. Usually, the gradient is smoothed using an order statistic filter with fixed coefficients chosen based on the gradient distribution. In practical applications, such as channel equalization, no prior information about the gradient exist and, therefore, an optimum filter for the gradient is usually unknown. In this paper, we implement an OSLMS adaptive algorithm that uses a modified adaptive *L*-filter to smooth the gradient in order to minimize the steady-state miss- adjustment for different noise distributions. The analysis of the new algorithm and the simulation results for the problem of channel equalization are presented. Comparison with other OSLMS algorithms shows the improvements of the new algorithm.

1. Introduction

Adaptive filters were applied with success in many areas of digital signal processing such as system identification, channel equalization, signal denoising, etc. The most familiar adaptive algorithm is the Least Mean Square (LMS) algorithm, due to its simplicity. However, the LMS adaptive filter is not suitable for applications in which the input and/or desired signals are corrupted by impulsive noise. In impulsive noise environments, the instantaneous gradient used to update the coefficients of the filter will have also an impulsive nature that can lead to an increase in the steady-state misadjustment. In order to deal with this problem, the class of Order Statistic LMS filters (OSLMS) was introduced [1], [2]. In the case of OSLMS filter the equation describing the updates of the filter coefficients is modified as follows:

$$\hat{\mathbf{h}}(n+1) = \hat{\mathbf{h}}(n) + \mu OS\{\mathbf{g}(n)\}\mathbf{a} \quad (1)$$

where $\hat{\mathbf{h}}(n)$ is the *N*x1 vector of adaptive filter coefficients, μ is the step-size, $\mathbf{a} = [a_1,...,a_L]^T$ is a column vector of weighting coefficients used to smooth the gradient. In (1) *OS{***g***(n)}* is the ordering operation applied to each row of the matrix **g***(n)*, and the **g***(n)* contains on the i^{th} row the past *L* values of the corresponding instantaneous gradient:

$$g(n) = \begin{bmatrix} e(n)x(n) & ... & e(n-L)x(n-L) \\ ... & ... & ... \\ e(n)x(n-N) & ... & e(n-L)x(n-N-L) \end{bmatrix} \quad (2)$$

where *x(n)* and *e(n)* are the input sequence and the output error. Depending on the selection of the weighting coefficients **a**, the class of OSLMS filters includes as members: average LMS (ALMS), median LMS (MLMS), Trimmed mean LMS (MxLMS) and Outer mean LMS (OxLMS).

2. The new algorithm

When the weighting coefficients are chosen properly, the OSLMS algorithms described in the last section can reduce the variance of the gradient estimate, and therefore, they have smaller steady-state excess mean square error. However, the selection of the weighting coefficients has to be based on some prior information about the gradient distribution. If the distribution of the gradient is not known a priori, an arbitrarily chosen smoothing filter will have poor performances. Therefore, in this paper we propose an algorithm that uses adaptive weighting coefficients **a***(n)* instead of **a** in (1) to smooth the gradient.

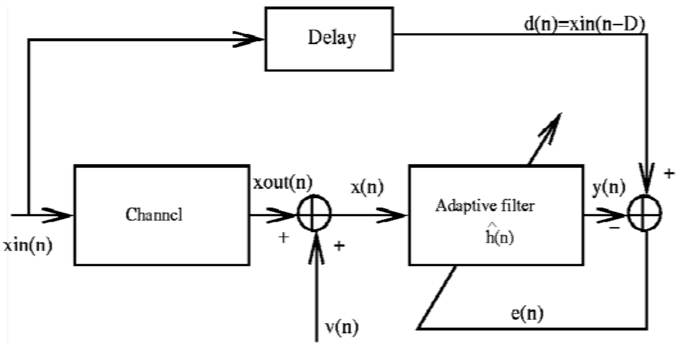

Fig 1 Channel equalization block diagram

In the case of the proposed Adaptive Order Statistic LMS (AOSLMS) algorithm there is no necessity to have any prior information about the gradient distribution since the weighting coefficients are continuously changed to adapt to the gradient. There are also in the literature some approaches that uses the adaptation of the weighting coefficients based on some statistic measurements of the gradient, but those approaches are limited to the modification of the trimming coefficient, such that the OS filter is modified between mean and median. The proposed algorithm is applied to the problem of channel equalization and the block diagram of the system used in this paper is depicted in Fig. 1, where a delayed version of the transmitted sequence through the channel $x_{in}(n)$ is used also at the receiver as the desired signal $d(n) = x_{in}(n - D)$.

To introduce the proposed AOSLMS algorithm, the updating equation (1) is modified as:

$$\hat{\mathbf{h}}(n+1) = \hat{\mathbf{h}}(n) + \mu OS\{\mathbf{g}(n)\}\mathbf{a}(n) \qquad (3)$$

where the notations are those from (1). We should note, that in the case of AOSLMS the values of the weighting coefficients **a***(n)* are not constants during the adaptation, but are adapted to the gradient distribution. In (3), the variable weighting coefficients **a***(n)* are applied to each row of the matrix OS{**g***(n)*} and the result is used to update the corresponding filter tap. In order to adapt the values of the weighting coefficients to the gradient distribution we have implemented an *L*-LMS filter. There are many publications dealing with the adaptive *L*-LMS filters, and it was already proved that these filters possess the ability to adapt their coefficients to the distribution of the input sequence (see [3], [4] and the references therein). The adaptation of the weighting coefficients **a***(n)* is done using samples of the gradient contained in the first row of **g***(n)* as depicted in Fig. 2, and the new algorithm is described as follows:

- compute the output *y(n)* and the error *e(n)* of the OSLMS filter:

$$y(n) = \hat{\mathbf{h}}(n)^T \mathbf{x}(n), \qquad e(n) = x_{in}(n-D) - y(n);$$

- update the vector of the weighting coefficients **a***(n)*:

$$\mathbf{a}(n) = \mathbf{a}(n) + \alpha \mathbf{g}_{(0)} e_g(n), \qquad (4)$$

where $\mathbf{g}_{(0)}(n)$ is the ordered version of the first row of **g***(n)*, and $e_g(n)$ is the error of the *L*-LMS filter applied to the gradient (see Fig. 2);

- update the coefficients of the OSLMS filter using (3).

In (4) we have used the error $e_g(n)$ to update the weighting coefficients. Since in this case there is no desired signal available for filtering the gradient we have chosen the constrained *L*-LMS described in [5], and (4) becomes:

$$\mathbf{a}(n) = \mathbf{P}\left[\mathbf{a}(n) + \alpha \mathbf{g}_{(0)} e_g(n)\right] + \mathbf{F}, \qquad (5)$$

where $y_a(n) = \mathbf{g}_{(0)}(n)\mathbf{a}(n)$ is the output from the *L*-LMS filter, and ***P*** and ***F*** are described in detail in [5] and [6]. In the case of AOSLMS algorithm there are basically two adaptive filters: one is used to adapt the weighting coefficients **a***(n)* of the gradient and the second one is the filter $\hat{\mathbf{h}}(n)$. The most sensible adaptive filter is **a***(n)* since if this filter does not converge also the AOSLMS filter will be divergent. Therefore, the step-size α has to be properly chosen in order to ensure its convergence for a wide range of gradient distributions. We have used a normalized *L*-LMS and the value of α in (5) is replaced by:

$$\alpha = \frac{\tilde{\mu}}{\gamma + \left\|\mathbf{g}_{(0)}(n)\right\|^2} \qquad (6)$$

The reason to use a step-size α given by (6) will be explained in the next section of the paper. Finally, the new algorithm is described by (3), (5), (6).

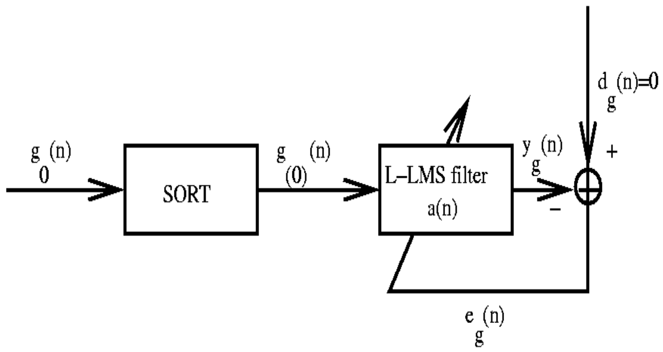

Fig 2 Block diagram of the L-LMS filter for gradient

3. Algorithm complexity and coefficients setup

The coefficients error vector is defined by (see [5]):

$$\Delta\mathbf{a}(n+1) = E\{\mathbf{a}(n+1) - \mathbf{a}_0\} = [\mathbf{I} - \alpha\mathbf{PRP}]\Delta\mathbf{a}(n). \qquad (7)$$

where $\mathbf{a}_0$ is the vector of the optimum coefficients, the matrix **P** is given in [5] and **R** is the covariance matrix of the input vector into the *L*-LMS filter, that is the ordered first row of the matrix **g***(n)*. From (7) we can see that the eigenvalues of the matrix **PRP** determine both the speed of convergence and the steady-state variance of the weighting coefficients **a**(n). In [5] it was proven that the condition to ensure the convergence of the weighting coefficients in the mean square error is:

$$0 < \alpha < \frac{1}{\sigma_{\max} + \frac{1}{2} tr[\mathbf{PRP}]}, \tag{8}$$

where $\sigma_{\max}$ is the maximum eigenvalue of the matrix **PRP** and $tr[\bullet]$ represents the trace of the matrix inside the brackets. If the step-size α is chosen to satisfy:

$$0 < \alpha < \frac{2}{3tr[\mathbf{R}]}, \tag{9}$$

then the condition (8) will be also satisfied and the convergence of the *L*-LMS is ensured (see [5] for more details). Therefore, if the step-size α in (5) satisfies the condition (9), for all input distributions, the weighting coefficients will converge to the optimum values. Since the input (the first row of the matrix **g**(n) is unknown, and therefore the trace of **R** is also unknown, a value for the parameter α that works well for any gradient distributions is difficult to find. In order to eliminate this problem, we have implemented a normalization of the step-size and the condition (9) becomes: $0 < \tilde{\mu} < 2/3$.

4. Simulations and Results

We have tested our algorithm in the channel equalization framework. The block diagram used in the experiments is depicted in Fig. 1. The new algorithm was compared with the median, trimmed mean and outer mean LMS algorithms. The length of the channel was $N_{ch} = 11$, the lengths of all the compared adaptive filters were *N=11* and the lengths of the weighting vectors **a** and **a**(n) for the gradient were *L=7*. The distribution of the channel noise *v(n)* has a generalized exponential density given by:

$$p(r) = k_1 e^{-k_2|r|^{\beta}}, \ |r| < \infty, \ 0 < \beta \le \infty, \ k_1 = \left(\beta k_2^{\ 1/\beta}\right)/2\Gamma(1/\beta), \ k_2 = [\Gamma(3/\beta)/\Gamma(1/\beta)]^{\beta/2} \sigma_v^{\ -\beta}, \tag{10}$$

Γ being the ordinary gamma function and σ_v the standard deviation. We have chosen the same step-sizes $\mu = 0.01$ for all the compared algorithms and the step-size for the *L*-LMS algorithm was $\tilde{\mu} = 0.1$. As β in (10) increases from a value close to zero, the resulting density varies from highly impulsive to Gaussian and to uniform. However, the gradient has a certain degree of impulsivity also in the case of Gaussian and uniform channel noise due to the desired signal *d(n)*. Therefore, we expect that the Outer Mean LMS do not give satisfactory results for any considered noise distribution. More than that, the gradient distribution is also influenced by the distribution of the channel noise and input sequence *x(n)* and the performance of the Median LMS is expected to be also poor. With these observations we can expect that among OSLMS algorithms with fixed weighting coefficients the Trimmed Mean LMS would have the best performance. Note that an algorithm similar to the proposed AOSLMS in which not only the trimming coefficient is adapted but also the envelope of the weighting coefficients would give even better results.

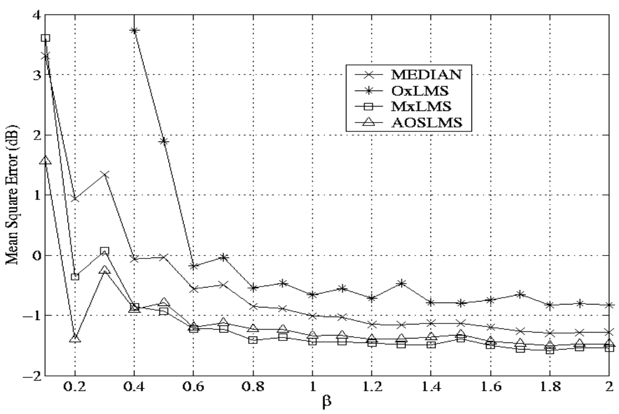

Fig. 3 Steady-state MSE for MLMS, MxLMS, OxLMS and AOSLMS for SNR=0dB.

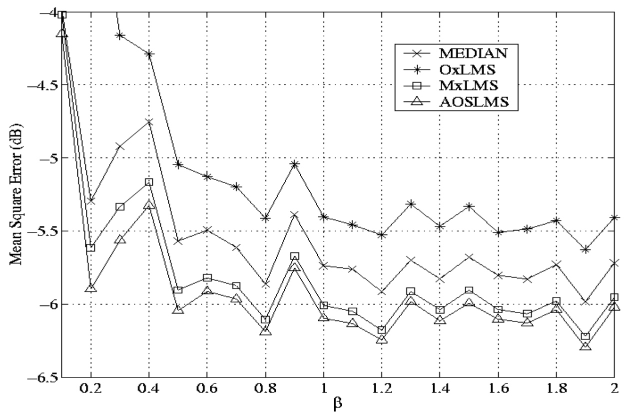

Fig. 4 Steady-state MSE for MLMS, MxLMS, OxLMS and AOSLMS for SNR=10dB

The simulations showing the performance of the AOSLMS filter compared with other OSLMS algorithms are given in Fig. 3 and Fig. 4 for different noise distributions. In these figures, the steady-state Mean Square Errors for each compared algorithms are plotted. In Fig. 3 the signal to noise ratio at the output of the channel was SNR=0dB, whereas in Fig. 4 the signal to noise ratio was SNR=10dB. From these figures, we can, see that the proposed algorithm gives better results for almost all considered noise distributions and this is in agreement with the theoretical considerations.

5. Conclusions

In this paper, we have applied a new AOSLMS adaptive filter to the problem of channel equalization for non-Gaussian noise environments. The approach of channel equalization differs from that of the system identification, in which the impulsive nature of the gradient is mainly given by the noise present in the system. Usually, in the case of channel equalization it is difficult to predict the distribution of the gradient and hence the optimal weighting vector to smooth the gradient. In such cases, the proposed AOSLMS algorithm would give better results due to its ability to adapt the weighting coefficients to the unknown gradient distribution.

References

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