MULTI-CHANNEL ADAPTIVE LEAST SQUARES LATTICE FILTERS FOR ACTIVE NOISE CONTROL

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ABSTRACT. In this paper a new algorithm for multi-channel adaptive least squares adaptive filtering, using a lattice filter is proposed. The algorithm is a multi-channel numerically stable version of the adaptive least squares lattice with a priori errors and error feedback. It is adapted to active noise control using the modified filtered-Error algorithm. It is shown throw computer simulations that the algorithm is stable, and that it has better convergence properties than algorithms based on the LMS and gradient adaptive lattice.[1]

INTRODUCTION

Active Noise Control has proven to be an effective technique for noise reduction in many applications. The FXLMS algorithm is the most popular algorithm for Active Noise Control, however it suffers from the same problem as the LMS algorithm, namely slow convergence and low sensitivity to eigenvalue spread [1][2]. For multi-channel systems, the MCFX-LMS algorithm is the most popular choice. It has additional problems of slow convergence when the input signals are highly correlated, which can greatly reduce the performance of some systems. The use of multi-channel recursive least squares algorithms (RLS), or Kalman filtering can solve this problems, however at the cost of a great increase of computational complexity [3][4]. Lattice implementations allow for a computational complexity of the order of $L \times N^2$ or $L \times N^3$ where *L* is the filter length and *N* is the number of input channels. The RLS has a computational complexity of the order of $(L \times N)^2$. In this paper a new multi-channel least squares adaptive lattice algorithm based on a priori errors with error feedback [2][5] is proposed, and adapted to active noise control. This implementation has good numerical properties allowing for the use of high order adaptive filters. The direct use of the filtered-X [6][7][8] structure for adapting multichannel algorithms to Active Noise Control requires the modification of the original algorithms, so in this paper the modified filtered error structure (Fe) [6] is used. The control filter is replaced by a lattice filter, allowing the simple replication of the adaptive lattice. The use of adaptive lattices in active noise control is not new. In [9] and [10], it was proposed the use of adaptive lattices based on the GAL algorithm and the DCFx structure for single channel active noise control. Chen [11] proposed and adaptation of the GAL algorithm to multichannel active control, but it only supported one reference signal, or multiple references combined into a single one. All this papers where based on the GAL algorithm. Bouchard [12] proposed the use of QR decomposition LSL, in a full multichannel system using the DCFx structure. However this requires the calculation of squares roots, or givens rotations, and has a greater computational complexity than the proposed algorithm. The proposed algorithm requires division.

MC-LSL ALGORITHM

The LSL algorithm can be viewed as an order recursive implementation of the RLS algorithm, based on the Levinson Durbin algorithm [13]. It pretends to solve the problem of given a set of reference signals,

$$\mathbf{u}(n)^T = [u_0(n)\, u_1(n) \cdots u_{N-1}(n)] \tag{1}$$

and of desired signals,

$$\mathbf{d}(n)^T = [d_0(n)\, d_1(n) \cdots d_{M-1}(n)] \tag{2}$$

determine the set of FIR filters, of order *L*, $\mathbf{w}_{i,j}(n)$, which minimize the weighted sum of the estimations errors,

$$\min_{\mathbf{w}} \left\{ \sum_{i=0}^{\infty} \lambda^i e_L^T(n-i) e_L(n-i) \right\}. \tag{3}$$

with,

$$\{\mathbf{e}_L(n)\}_j = d_j(n) - \sum_{i=0,j=0}^{N,M} \mathbf{w}_{i,j}^T \mathbf{u}_i(n:N-L+1). \tag{4}$$

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where $\{a\}_i$ represents the *i* element of the vector *a*, and,

$$\mathbf{u}_i(n:N-L+1)^T = [u_i(n)\cdots u_i(n-L+1)] \tag{5}$$

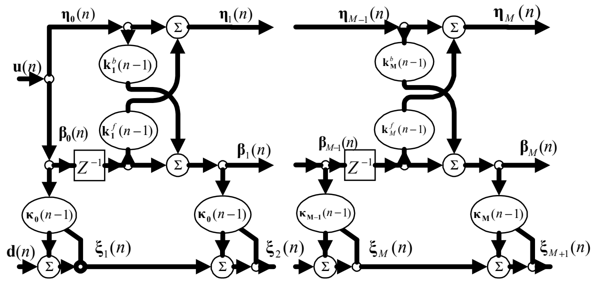

Fig. 1. A lattice filter/

The solution of this equations leads to the recursive least squares algorithm. However this algorithm has a computational complexity which grows with L^2. The Least Square Lattice Algorithms uses an order recursive approach resulting in a computational complexity which varies with *L*. A multi-channel version of the LSL with error feedback is presented in table 2. In figure 1 is presented the lattice filter associated with the algorithm. This can be derived in a similar way to the ones in [5] and [14].

Variable	Meaning
$\boldsymbol{\eta}_P(n)$	a priori forward prediction error vector
$\boldsymbol{\beta}_P(n)$	a priori backward prediction error vector
$\mathbf{F}_P(n)$	autocorrelation matrix of the forward prediction errors
$\mathbf{B}_P(n)$	autocorrelation matrix of the backward prediction errors
$\gamma_P(n)$	conversion factor
$\boldsymbol{\xi}_P(n)$	a priori joint process estimation error

Table 1. Least Squares Lattice Variables

Table 1 defines the variables used in the algorithm. The a posteriori prediction and estimation errors, $\mathbf{f}_P(n)$, $\mathbf{b}_P(n)$ and $\mathbf{e}_P(n)$, are related to the a priori prediction and estimation errors, $\boldsymbol{\eta}_P(n)$, $\boldsymbol{\beta}_P(n)$ and $\boldsymbol{\xi}_P(n)$, by,

$$\mathbf{f}_p(n) = \gamma_P(n-1)\boldsymbol{\eta}_P(n) \tag{6}$$

$$\mathbf{b}_p(n) = \gamma_P(n)\boldsymbol{\beta}_P(n) \tag{7}$$

$$\mathbf{e}_p(n) = \gamma_P(n-1)\boldsymbol{\xi}_P(n) \tag{8}$$

Initialization

$$\boldsymbol{\eta}_0(n) = \boldsymbol{\beta}_0(n) = \mathbf{u}(n)$$

$$\boldsymbol{\xi}_0(n) = \mathbf{d}(n)$$

$$\mathbf{F}_{P-1}(-1) = \mathbf{B}_{P-1}(-1) = \delta$$

$$\mathbf{K}_P^f(0) = \mathbf{K}_P^b(0) = 0$$

$$\gamma^{-1}(n) = 1$$

$$\gamma \text{ small}$$

for p=1 to L

Prediction

$$\mathbf{F}_{P-1}(n) = \lambda \mathbf{F}_{P-1}(n-1) +$$
$$\boldsymbol{\eta}_{p-1}(n)\boldsymbol{\eta}_{p-1}(n)^T \gamma_{P-1}(n-1)$$
$$\mathbf{B}_{P-1}(n-1) = \lambda \mathbf{B}_{P-1}(n-2) +$$
$$\beta_{P-1}(n-1)\beta_{P-1}(n-1)^T \gamma_{P-1}(n-1)$$
$$\boldsymbol{\eta}_P(n) = \boldsymbol{\eta}_{P-1}(n) + \mathbf{K}_p^f(n-1)\boldsymbol{\beta}_{P-1}(n-1)$$
$$\boldsymbol{\beta}_P(n) = \boldsymbol{\beta}_{P-1}(n) + \mathbf{K}_p^b(n-1)\boldsymbol{\eta}_{P-1}(n)$$
$$\mathbf{K}_P^f(n) = \mathbf{K}_P^f(n-1) -$$
$$\boldsymbol{\eta}_P(n)\boldsymbol{\beta}_{P-1}(n-1)^T \mathbf{B}_{P-1}^{-1}(n-1)\gamma_{P-1}(n-1)$$
$$\mathbf{K}_P^b(n) = \mathbf{K}_P^b(n-1) -$$
$$\boldsymbol{\beta}_P(n)\boldsymbol{\eta}_{P-1}(n)^T \mathbf{F}_{P-1}^{-1}(n)\gamma_{P-1}(n-1)$$
$$\gamma_P(n-1) = \gamma_{P-1}(n-1) -$$
$$\boldsymbol{\beta}_{P-1}^T(n-1)\mathbf{B}_{P-1}^{-1}(n-1)\boldsymbol{\beta}_{P-1}(n-1)$$
$$\gamma_{P-1}^2(n-1)$$

Filtering

$$\boldsymbol{\xi}_P(n-1) = \boldsymbol{\xi}_{P-1}(n-1) - \mathbf{K}_{m-1}(n-2)\boldsymbol{\beta}_{P-1}(n-1)$$
$$\mathbf{K}_{P-1}(n-1) = \mathbf{K}_{P-1}(n-2) +$$
$$\boldsymbol{\xi}_P(n-1)\boldsymbol{\beta}_{P-1}(n-1)^T \mathbf{B}_{P-1}^{-1}(n-1)$$
$$\gamma_{P-1}(n-1)$$

Table 2. A posteriori error adaptive least squares lattice filter algorithm.

For a system with a high number of channels the computational complexity of the algorithm can still be reduced by using the matrix inversion lemma (eq. 12, [14]) to propagate the inverse of the auto-correlation matrix instead of the autocorrelation matrix itself.

$$A = B + CDC^H \Leftrightarrow$$
$$A^{-1} = B^{-1} - B^{-1}C(D^{-1} + C^H B^{-1} C)^{-1} C^H B^{-1} \qquad (10)$$

This gives the following recursion for the updates,

$$\lambda \mathbf{B}_{m-1}^{-1}(n) = \lambda \mathbf{B}_{m-1}^{-1}(n-1) - \frac{\mathbf{B}_{m-1}^{-1}(n-1)\boldsymbol{\beta}_{m-1}(n)(\boldsymbol{\beta}_{m-1}(n))^T \mathbf{B}_{m-1}^{-1}(n-1)\gamma_{m-1}(n)}{\lambda + (\boldsymbol{\beta}_{m-1}(n))^T \mathbf{B}_{m-1}^{-1}(n-1)\boldsymbol{\beta}_{m-1}(n)\gamma_{m-1}(n)} \qquad (11)$$

$$\lambda \mathbf{F}_{m-1}^{-1}(n) = \lambda \mathbf{F}_{m-1}^{-1}(n-1) - \frac{\mathbf{F}_{m-1}^{-1}(n-1)\boldsymbol{\eta}_{m-1}(n)(\boldsymbol{\eta}_{m-1}(n))^T \mathbf{F}_{m-1}^{-1}(n-1)\gamma_{m-1}(n-1)}{\lambda + (\boldsymbol{\eta}_{m-1}(n))^T \mathbf{F}_{m-1}^{-1}(n-1)\boldsymbol{\eta}_{m-1}(n)\gamma_{m-1}(n-1)} \qquad (12)$$

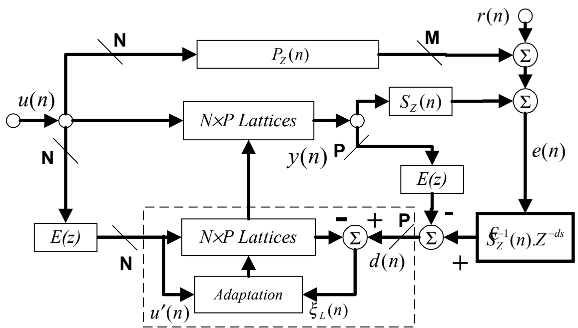

Fig. 2. Adaptation of the MC-LSL algorithm for Active Noise Control using the delay compensated filtered-Error structure.

DELAY COMPENSATED FILTERED-ERROR STRUCTURE

The algorithm presented can't deal with the effects of the cancellation path. However it is possible to adapt the algorithm for use in active noise control using the structure presented in figure 2. This is a combination of the modified filtered-X structure or delay compensated filtered-X [7][8] and de filtered Error structure[6], resulting in the delay compensated filtered Error structure, DCFe. In [6] the filter *E*(*z*) is taken as a pure delay. In the multichannel case it makes sense to generalize the algorithm to take an arbitrary *E*(*z*). This makes it easier to invert the secondary path. *E*(*z*) should include the frequency responses of elements common to all the channels, namely anti-aliasing filters and reconstruction filters. When the cancellation path estimate is equal to the primary path, this technique allows the removal of the cancellation path and the simple implementation of the LSL algorithm. It is possible to generalize the results in [7], to show that for arbitrary delay in the secondary the algorithm still converges even with error in the estimate of $S^{-1}(z)E(z)$, the secondary path inversion. This as long as the phase errors are bellow 60º and amplitude is greater than twice the correct one.

SIMULATION RESULTS

In this section some results of computer simulations are presented. The simulations serve as a demonstration of the FeMcLsl algorithm and as a comparison with other known algorithms.

The transfer functions of the several signal paths, namely secondary path and primary path where modeled as delay followed by second order system, matching the resonant frequency of the (1,1,1) mode of an acoustic chamber with dimensions 251 *cm* × 125 *cm* × 125 *cm*. This corresponds to 203 *Hz*. The damping factor was made 0.01, and the sampling frequency was 1000 *Hz*. This results that the z-transform of the impulse response of the paths where,

$$\frac{Az^{-d}}{2.6554 - 2.0255\,z^{-1} + 1.0000\,z^{-2}} \qquad (13)$$

where *A* changed from 0.5 to 2, and *d* changed from 1 to 20, according with the position of the reference microphone and anti-noise sources.

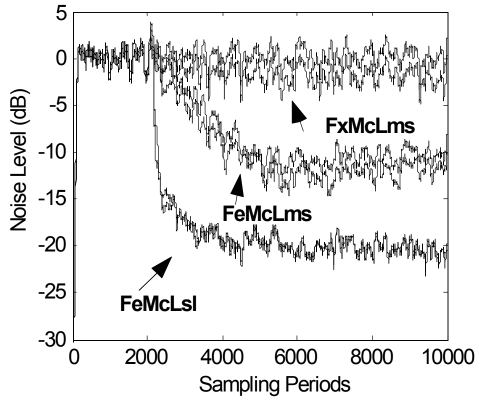

Fig. 3. Comparison of the Learning curves of the FxMcLms, FeMcLms and FeMcLsl algorithm, for a 2 × 2 × 2 system.

The simulations correspond to 2 × 2 × 2 systems, with two references, two anti-noise sources and two error microphones. It can be seen that the proposed algorithm clearly outperforms the multichannel Fx-LMS and the multichannel Fe-LMS. It also outperforms the multichannel version of the gradient adaptive lattice algorithm (McGal).

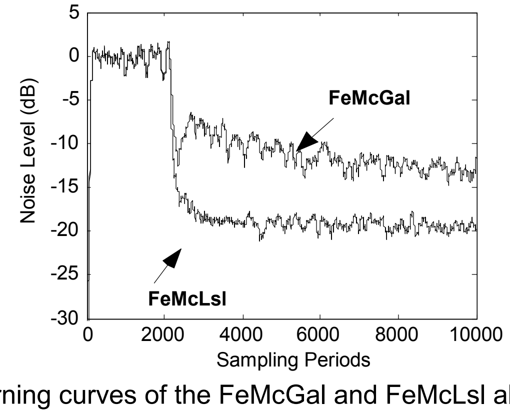

Fig. 4. Comparison of the Learning curves of the FeMcGal and FeMcLsl algorithm, for a 2 × 2 × 2 system.

CONCLUSION

A multi-channel recursive least squares lattice algorithm, which is suitable to use in active noise control was presented. It was adapted to active noise control using the McFe structure, resulting in a functioning system. Simulation results show that the proposed algorithm outperforms other algorithm based on the LMS or in the GAL, especially for the case of broadband multi-channel systems with highly correlated reference signals.

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