A LINEAR ALGORITHM FOR THE NOISE SHRINKAGE IN COMMUNICATION SYSTEMS WITH GAUSSIAN NONSTATIONARY INPUTS

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ABSTRACT. In this paper, we consider digital communication systems with Gaussian input signals and bandpass nonlinearities. We suggest a linear algorithm that being applied at the receiver allows noise caused by the nonlinear channel be reduced. The work is based on the Bussgang's theorem generalisation for the Gaussian nonstationary inputs. The examples of the application of the proposed technique in the CDMA system are given.

1. INTRODUCTION

Modern digital communication systems widely employ signals that have a multi-component structure. Such are, for example, the orthogonal frequency division multiplexing (OFDM) or CDMA signals (in the down-link). Really, both of them are generated as a sum of independent components, in the former case the signal is obtained by the application of the inverse discrete Fourier transform to the input information symbols while the latter results from summing the spread signals of different users.

Just due to the multi-component structure the above signals are largely subjected to the nonlinear distortions from the high power amplifiers (PA). Because of the nonlinear behaviour of the PA with respect to the signals with envelope fluctuations, the analysis and compensation of the nonlinear effects is very important in the communication systems that employ multi-component signals.

Fortunately, if the Gaussian distribution of the input signal is acceptable (i.e. the number of the component in the resulting signal is large enough for the validity of the central limit theorem [1]) then the nonlinear effects can be estimated analytically on the basis of the generalisation of the Bussgang's theorem presented in [2].

In this paper, on the basis of the results presented in [2] as well as their development given in [3] we propose a linear algorithm for noise reduction. By employing the estimates obtained in [2], [3] we prove that the actual noise power that affects making a decision can be decreased, that is after the proposed procedure the actual signal-to-noise power is augmented.

2. MODEL OF THE SYSTEM

A baseband part of the system under consideration is shown in Fig.1. The receiver filter is matched to the transmitted pulse. Generally speaking, the scheme in Fig.1 is more suitable for the presentation of the OFDM system but it is also acceptable for the CDMA system when all users have the same chip waveform.

In both cases (for the sake of simplicity we concentrate on the case of synchronous CDMA) the baseband signal at the input of the non-linear block can be described as:

$$i(t) = \sum_{m=-\infty}^{\infty} \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} c_{n,k}\, g(t - nT - mNT)\,, \quad (1)$$

where K is a number of components of the multiplex signal,

m is the time index,

n is the sample number in the m-th signal symbol,

$c_{n,k}$ jointly describes the modulation and code for the k-th component,

NT is the symbol time,

$g(t)$ specifies the impulse characteristic of the transmitter shaping filter, its Fourier transform is $T(f)$ (see Fig.1). In the CDMA system $g(t)$ describes the chip waveform.

For instance, for the OFDM signal N and K equal to the number of the subcarriers, for the CDMA signal K is the number of the users while N is the length of the spreading sequence.

It has been proven in [2] that when K is large enough (1) describes a nonstationary Gaussian signal with non-zero mean (provided that the input information symbol is fixed to the specific value). Presented in [2] theorems give an analytical framework for the estimation of the transformation of $i(t)$ by the nonlinear channel into the received signal $u(t)$ (see Fig.1):

$$u(t) = K_0 i(t) + d(t) + w(t)\,. \quad (2)$$

In (2) K_0 is the attenuation coefficient,

$d(t)$ is the uncorrelated with $i(t)$ noise caused by the nonlinearity and $w(t)$ is the channel noise.

Both the attenuation coefficient and the nonlinear noise power depend on the transfer characteristic of the nonlinearity as well as the average power of the input signal [2]. The power of the input signal and the type of the nonlinear characteristic determine the operating point of the nonlinear device. The output back-off (OBO) is commonly used in order to specify the operating point. OBO is defined as the difference (in dB) between the output saturation power and average output power of the nonlinear device.

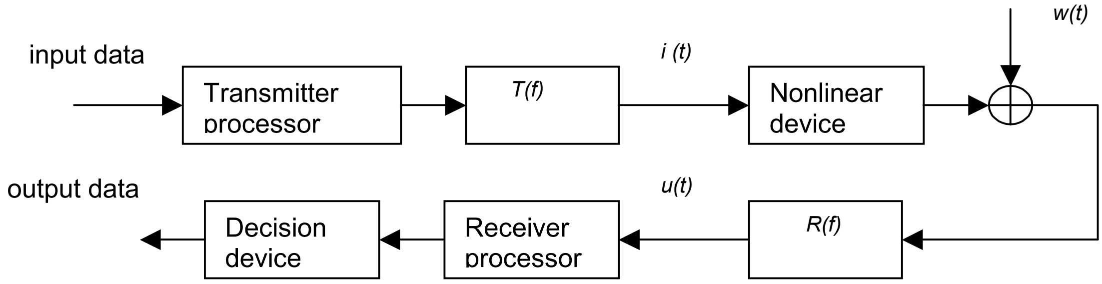

Fig.1 Block-diagram of the base-band part of the communication system

Usually, the rotation that is equal to $\arg\{K_0\}$, is compensated before the decision device and in the further analysis we will take into account only the attenuation that is equal to $|K_0|$.

The following relationship is valid:

$$P_o = |K_0|^2 P_{in} + P_d \quad , \tag{3}$$

where P_o and P_{in} are , respectively, the average powers of the output and input bandpass signals, P_d is the average power of the bandpass nonlinear noise. We assume that the channel noise is characterised by the variance $E\{w(t)w^*(t)\} = 2P_w$.

We assume that the complex transfer characteristic of the nonlinear block is described by the function $S(x) = F_A(x)e^{jF_P(x)}$,where x is the amplitude of the input signal, $F_A(x)$ and $F_P(x)$ specify, respectively, the amplitude modulation-to-amplitude modulation (AM/AM) and amplitude modulation-to-phase modulation (AM/PM)conversions. If the input signal can be considered as Gaussian, then

$$P_o = \frac{1}{2}\int_0^\infty \frac{x}{P_{in}} e^{-x^2/2P_{in}} F_A^2(x)dx .$$

We consider two models of the PA. The first one is the model of the travelling wave tube amplifier (TWTA) [2] with

$$F_A(x) = \frac{A_{sat}^2 x}{x^2 + A_{sat}^2}, F_p(x) = \frac{\pi}{3}\frac{x^2}{x^2 + A_{sat}^2} . \tag{4}$$

The second model considered is that of soft envelope limiter that is convenient for the description of the ideally predistorted amplifier (IPA). For this model AM/AM and AM/PM conversions are described by:

$$F_A(x) = \begin{cases} x, & x \le A_{sat.} \\ A_{sat.}, & x > A_{sat.} \end{cases}, F_p(x) = 0 . \tag{5}$$

In (4) and (5) $A_{sat.}$ is the amplifier input saturation voltage.

It was proved in [2], [4] that (2) is valid for the OFDM systems with rectangular pulse shapes as well as for the CDMA systems with rectangular chip waveforms and synchronous CDMA with *K=N*.

If the performance of the system is assessed in terms of the bit-error probability (*BEP*), the actual signal-to-noise ratio that affects the decision should be estimated. Therefore, the further analysis of the system is defined by the specific kind of the signal and modulation used. Usually, $u(t)$ specified by (2) is used for further processing. It means that for the estimate $i(t)$ its observation defined by (2) is taken, that is

$$\hat{i}(t) = u(t)/|K_0| , \tag{6}$$

where $\hat{i}(t)$ is the estimate of $i(t)$.

In order to ensure the validity of the obtained in [2],[4] results we consider the systems with rectangular pulse shaping and sufficiently large *K* .

Below we present the algorithm that being applied to $u(t)$ (or to its samples) leads to increasing the real signal-to-noise ratio.

3. DESCRIPTION OF THE PROPOSED ALGORITHM

If for the estimation of *i(t)* the procedure (6) is employed then the variance of the estimation error is:

$$E\{(i(t)-\hat{i}(t))(i(t)-\hat{i}(t))^*\} = \frac{2P_d + 2P_w}{|K_0|^2}. \qquad (7)$$

If we are interested in linear 'denoising algorithms' then our purpose is to find the estimate $\hat{i}(t)$ in the form of

$$\hat{i}(t) = Ru(t) \qquad (8)$$

in order to minimise the variance of the estimation error $\sigma^2_{(i-\hat{i})}(t) = E\{(i(t)-\hat{i}(t))(i(t)-\hat{i}(t))^*\}$. In (8) R is a constant.

On the basis of (2), (3) it is easy to find that

$$R = \left\{P_{in}\frac{|K_0|}{|K_0|^2 P_{in} + P_d + P_w}\right\} = \left\{P_{in}\frac{|K_0|}{P_0 + P_w}\right\}. \qquad (9)$$

When (8), (9) are used the variance of the estimation error is:

$$E\{(i(t)-\hat{i}(t))(i(t)-\hat{i}(t))^*\} = \frac{2P_{in}P_o(P_d + P_w)}{(P_0 + P_w)^2}. \qquad (10)$$

By comparison of (7) and (10) it is easy to show that that the proposed procedure decreases the variance of the estimation error for any values of P_{in}, P_o and P_w.

4. COMPARISON OF THE CONVENTIONAL AND PROPOSED SYSTEMS

After filtering the received signal (2) is sampled at instants $t_{m,n} = nT + mNT$. Therefore, by introducing the discrete time we have

$$\mathbf{u}(m) = K_0\mathbf{i}(m) + \mathbf{d}(m) + \mathbf{w}(m),$$

where $\mathbf{u}(m)$, $\mathbf{i}(m)$, $\mathbf{d}(m)$ and $\mathbf{w}(m)$ are the *N*-dimensional vectors and $\mathbf{u}_n(m) = u(t_{m,n})$,

$\mathbf{i}_n(m) = i(t_{m,n})$, $\mathbf{d}_n(m) = d(t_{m,n})$, $\mathbf{w}_n(m) = w(t_{m,n})$.

Then the sequence $\mathbf{u}(m)$ is processed either by the discrete Fourier transform block (in OFDM systems) or by the despreader (in CDMA systems).

In order to assess BEP it is necessary to specify the type of the signal and modulation. We concentrate on the synchronous CDMA system that uses Walsh orthogonal codes with $N = 64$, $K = N$ and with 4-QAM signals. Then, for conventional processing the BEP is:

$$P_b = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{|K_0|^2}{(\sigma_D^2 + \sigma_W^2)}}, \qquad (11)$$

where $\frac{|K_0|^2}{(\sigma_D^2 + \sigma_W^2)} = SNR$ is the actual signal-to-noise ratio before the decision device and σ_D^2 and σ_W^2 are the actual variances of noise (related to P_{in}) caused, respectively, the nonlinearity and the channel. Evaluation of σ_D^2 and σ_W^2 is given in [2], [4] and here we report the results:

$\sigma_D^2 = \frac{2P_d}{P_{in}}$, $\sigma_W^2 = \frac{N_0 P_0}{E_b P_{in}}$, where $\left(\frac{E_b}{N_0}\right)$ is the ratio of the signal power per bit to the spectral density of the channel noise.

At the same time, after the application of the proposed procedure (8), (9) the actual signal-to-noise ratio becomes:

$$SNR_{\text{mod.}} = \frac{(|K_0|^2 + \frac{\sigma_D^2 + \sigma_w^2}{2})}{\sigma_D^2 + \sigma_w^2} = SNR + 0.5. \qquad (12)$$

Therefore, the proposed procedure leads to the enhancement of the actual *SNR*. This enhancement is observed for any type of the modulation because of decreasing the variance of the estimation error of $i(t)$. However, the specific value of the actual *SNR* depends on the type of the modulation used. For the system under consideration the enhancement is described by (12).

In Fig.2 we present the graphs of BEP for the above system with TWTA with and without application of the proposed procedure.

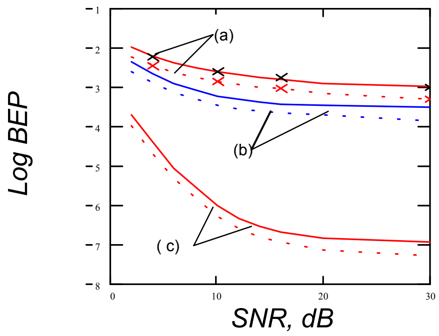

Fig.2 Bit-error probability for the system with TWTA and different OBO values: (a)- OBO=1.95, (b)- OBO=2.45, (c)- OBO=3.96. Single points correspond to the simulation results. Solid lines present the estimates for conventional processing, dotted lines – for suggested processing.

In Fig.3 the graphs of BEP versus OBO are shown for the case when the signal is distorted only by the nonlinearity (i.e. $SNR = \infty$). It is seen that the obtained improvement of the BEP depends on the OBO slightly.

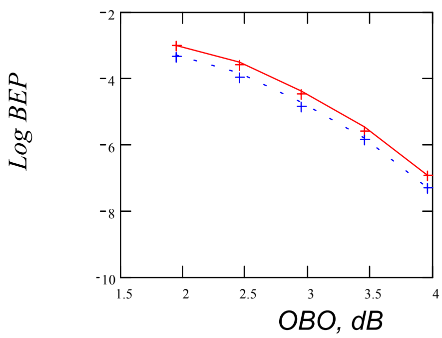

Fig.3 Asymptotic bit-error probability $P_{b\infty}$. Solid lines correspond to conventional processing, dotted lines - to suggested processing. Single points present the simulation results

The optimum operating point of the PA can be defined on the basis of the analysis of the total degradation (TD) of the PA [5]. The TD is specified for the fixed *BEP=BEP₀* as:

$$TD = (\frac{E_b}{N_0})_{nonlin.} - (\frac{E_b}{N_0})_{lin.} + OBO \quad (dB),$$

where $(\frac{E_b}{N_0})_{nonlin.}$ and $(\frac{E_b}{N_0})_{lin.}$ are the required SNR for obtaining BEP_0 when, respectively, the nonlinear and linear amplifiers are used.

An analytical evaluation of the *TD* in the communication systems with Gaussian input signals and different kinds of the nonlinear PA is presented in [3]. By combining the estimates obtained in [3] with (12) we can find that decreasing in *TD* after the application of the proposed procedure is:

$$\Delta TD = 10\log\left(\frac{\gamma\{1-(2\gamma-1)\theta\}}{(1-2\gamma\theta)(\gamma-1/2)}\right) \quad (dB),$$

where $\theta = \frac{P_0}{|K_0|^2 P_{in}} - 1$ and γ is the actual *SNR* required to provide BEP_0.

The graphs of TD for conventional processing as well for proposed one for the systems with IPA and TWTA are presented in Fig.4.

5. CONCLUSIONS

In this paper, we consider the digital communication systems with Gaussian input and bandpass nonlinearity. A linear signal-processing procedure for noise shrinkage is proposed. It is based on the generalization of the Bussgang's theorem for the nonstationary Gaussian inputs [2]. The procedure consists in the multiplication of the received signal (or its samples) by the specially calculated gain (instead of the gain that is employed conventionally). The conventional and modified gains depend on the same parameters that are known at the stage of the system design.

The analytical estimates for bit-error probability and total degradation have been derived and compared with those obtained in the conventional systems.

Because of the simplicity and not very significant improvement in *SNR* the proposed algorithm can be applied at the first stage of 'de-noising' the received signal.

The proposed system has been compared with the conventional one analytically and via the simulations in the terms of the bit-error probability and total degradation. The results for the CDMA system with TWTA and IPA and with 4-QAM signals are presented.

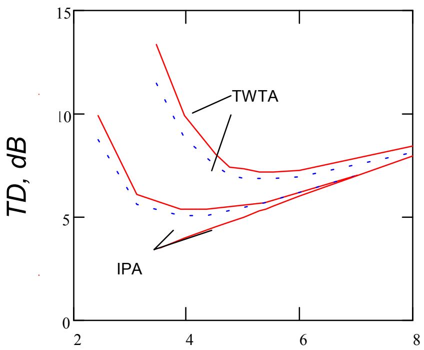

Fig.4 Total degradation in the systems with TWTA and IPA. Solid lines correspond to conventional processing, dotted lines present TD when the proposed algorithm is used.

6. REFERENCES

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