EXACT SPHERE CONSTRAINED MAXIMUM LIKELIHOOD DETECTION OF PARAMETERS IN CDMA

Khan E.

Mobile Communications Department, Institute Eurecom
B.P. 193, 06904, Sophia-Antipolis Cedex, France, E-mail: Ejaz.Khan@eurecom.fr

Abstract

We describe constrained Maximum likelihood (ML) detection technique for the CDMA. We consider exact sphere constrained ML problem. In this detection scheme the detected data vector is constrained to lie on the sphere. In [1], the authors solved the ML problem under the constraint that the detected vector lie within the hypercube and named it as box constrained ML. They also proposed the maximization of the likelihood function over sphere, i.e., confine the solution vector to lie within the sphere and project the solution vector onto sphere. In fact, in the sphere constrained problem the solution vector lie on the sphere and not in the interior of the constraining sphere (as is done in [2]). The other problem with their method is that the small error in the solution vector can cause large error when projected onto sphere (provided that the solution vector lie well inside sphere). The goal of this paper is to solve the sphere constrained problem exactly.

Signal Model

In this section we describe the base band uplink model of the CDMA communication system. We consider asynchronous CDMA with single path channels. The signal is corrupted by the presence of the additive white Gaussian noise (AWGN) with zero mean and variance $\frac{N_\circ}{2} = \sigma^2$. The number of the users in the system are assumed to be K. The processing gain, $N = \frac{T_d}{T_c}$, where T_d is the symbol duration and T_c is the chip duration. The users transmit binary information symbol stream $d_k(n) \in \{-1,1\}$, $n = 0,1,........,L-1$, is the symbol interval index and L is the length of the data block. $s_k(n) = (s_k(nN+1).........s_k(n+1)N)^T$, with $s_k(i) \in (-\frac{1}{\sqrt{N}}, \frac{1}{\sqrt{N}})$ is the spreading code of user k to modulate nth bit. When users are allowed random access to the channel, each user encounters a transmission delay relative to other users. We assume that all delays are integer multiple of the chip duration.

The continuous-time received signal is down converted to the base band, passed through the chip pulse shape and sampled. The received signal after chip matched filtering is then

$$r = \sum_{n=0}^{L-1} \sum_{k=1}^{K} d_k(n) c_k(n) + v \qquad (1)$$

where $c_k(n) = [0^T_{nN+\tau_k} \, s_k(n) 0^T_{(L-n)N-\tau_k-1}]^T$,

v is a vector of independent, identically distributed Gaussian random variable of zero mean and variance $\sigma^2 = \frac{N_\circ}{2}$. The above equation can also be written in matrix form

$$r = Sd + v$$

where the data symbol vector is given by $d = (d_1(0), d_2(0)........d_K(L-1))^T = (d_1, d_2,.......d_{LK})^T$

the minimal set of sufficient statistics of dimension LK is obtained through correlation, matched to the received signal. This also ensures maximization of SNR, i.e.,

$$y = S^T r = S^T Sd + S^T v = Rd + z \qquad (2)$$

where R is correlation matrix and is positive definite with probability one, z is zero mean Gaussian vector with variance $\sigma^2 R$.

Exact sphere constraint ML

Given the set of data $y \in R^{LK}$, our goal is to find symbol vector that maximizes the likelihood function or minimizes the negative of it. The negative of the loglikelihood function is given by

$$l(d) = d^T Rd - 2y^T d \quad (3)$$

We maximize loglikelihood equation subject to the constraint that the symbol vector lie on the sphere. This is constrained optimization problem and can be solved using Lagrange multiplier method.

The Lagrangian is as follows

$$L(d, \lambda) = d^T Rd - 2y^T d + \lambda(d^T d - LK) \quad (4)$$

To calculate the stationary points, we differentiate with respect to d and λ; setting these partial derivatives equal to zero we have

$d = (R + \lambda I)^{-1} y$ and $d^T d = LK$.

These are known as Lagrange equations. Now the problem is to solve the above equations for the Lagrange multiplier, i.e., λ . Using the Lagrange equations we have

$$f(\lambda) = y^T (R + \lambda I)^{-2} y - LK \quad (5)$$

The optimal value of the Lagrange multiplier λ^* is the zero of the above equation. We proceed by computing the eigenvector decomposition to diagonalize equation (5), i.e.,

$$f(\lambda) = y^T (U\Sigma U^T + \lambda I)^{-2} y - LK$$

where $R = U\Sigma U^T$ is the eigenvector decomposition of matrix R. After bit of algebra we obtain the following equation

$$f(\lambda) = \lambda^{-2} \sum_i \frac{z_i^{\ 2}}{(\lambda\sigma_i + 1)^2} - LK \quad (6)$$

where $z = U^T y$ and z_i is the i^{th} component of vector z . σ_i are the eigenvalues of matrix R . Equation (6) is nonlinear and can be solved numerically using Newton-Raphson method to obtain the value of λ^* . There may be many solutions of equation (6). The value of λ^* which maximizes the likelihood must be chosen. Initial simulations shows promising results.

References

1) P.H. Tan, L. K. Rasmussen and T. J. Lim, “Box-constrained maximum likelihood detection in CDMA”, in the proceedings of the Zurich Seminar on Broad band Comm. , pp. 55-62, Feb. 2000.
2) P.H. Tan, L. K. Rasmussen and T. J. Lim, “Sphere-constrained maximum likelihood detection in CDMA”, in the proceedings of IEEE Veh. Tech. Conf. (Tokyo, Japan), pp. 517-521, May 2000.
3) P.H. Tan, L. K. Rasmussen and T. J. Lim, “Constrained maximum likelihood detection in CDMA”, IEEE Trans. Commun. , vol. 49, pp. 142-153, Jan. 2001.