JOINT DELAY AND ANGLE OF ARRIVAL ESTIMATION USING EM ALGORITHM FOR MULTIPATH SIGNALS ARRIVING AT ANTENNA ARRAY

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Abstract

We propose a novel approach to estimate the angles of arrival (AoA) and delays of the multipath signals from digitally modulated source arriving at an antenna array. The method avoids computationally expensive optimization search by using Expectation Maximization (EM) algorithm.

Wireless communications channel can be characterized by slow varying and fast varying parameters. The amplitudes and the relative phase of each path are fast varying and is subject to (Rayleigh) fading. The stationarity of the fading depends on the speed of the mobile. Our method exploits the stationarity of the angles and delays, as well as the independence of fading over slots. We assume that antenna array response has a known structure.

Data Model

Consider a single user case which transmit signal via multipaths. The received base band signal at an M- element antenna array at the time t in the nth interval is

$$y^n(t)=\sum_l S_l^n h^n(t-lT)+n^n(t) \quad (1)$$

where T is the symbol period and $n^n(t)$ is the additive white Gaussian noise. S_l^n is the digital transmitted sequence. $h^n(t)$ is the channel. The channel can be modeled as

$$h^n(t)=\sum_{i=1}^{Q} a(\theta_i)\beta_i(n)g(t-\tau_i) \quad (2)$$

Where Q are the total number of paths. Each path is parameterized by the direction of arrival (DOA), θ_i, time delay τ_i and the complex path attenuation (fading), $\beta_i(n)$. The array response to the path from direction θ_i is $a(\theta_i)$ and g(t) is known pulse shape (modulation waveform). After sampling at a rate P times the symbol rate. A block formulation is obtained by taking N snapshot, yields [1],

$$Y^n = H^n S^n + N^n, \ n=1,2,....,S \quad (3)$$

The first step is to estimate the channel impulse response from the user to the antenna array. This can be accomplished by using training sequence or using some blind method. Let $\overline{H^n}=X(n)$ be the estimate of the true channel H^n. We have

$$\overline{H^n}=X(n)=H^n+V^n \quad (4)$$

where V^n is the estimation noise matrix. H^n can be written as

$$H^n = A(\theta)diag(\beta(n))G^T(\tau) \quad (5)$$

where, $A(\theta)=[a(\theta_1),a(\theta_2).........a(\theta_Q)]$, $\beta(n)=[\beta_1(n).......\beta_Q(n)]$ and $G^T(\tau)=[g^T(\tau_1)........g^T(\tau_Q)]$.

Applying vectorization operator to equation (4) and using equation (5) and writing the resulting equation yields

$$x(n)=U(\theta,\tau)\beta(n)+v(n), \ n=1,2,....,N \quad (6)$$

where

$$U(\theta,\tau)=G(\tau)\circ A(\theta)=[u(\theta_1,\tau_1),.....u(\theta_Q,\tau_Q)] \quad (7)$$

where $\circ$ denotes column-wise Kronecker product. $u(\theta,\tau)$ is the space time matrix. It is assumed that the structure of the space time matrix U is known. It is also assumed that the path fadings are normally distributed with zero mean. Estimation noise V, is white Gaussian. To be able to identify (θ,τ) using equation (6), we need $U(\theta,\tau)$ to be strictly tall and full column rank.

EM based Joint angle and delay estimation (JADE)

EM is an iterative method of likelihood estimation. It consists of two steps: 1) Expectation step and, 2) Maximization step and makes use of augmented data set known as complete data set to make the ML problem easy.

Given equation (6), we are now ready to define the complete data set. Let $z = (x(n), \beta(n))$ be complete data, where $x(n), \beta(n)$ are the estimated channel and fading coefficients (hidden data) respectively. The pdf of z as function of u can be written as the product between the likelihood and priori distribution of β , i.e.,

$$f(z;u) = f(x(n), \beta(n); u) = f(x(n)/\beta(n), u) f(\beta(n); u) \qquad (8)$$

Having multiple channel estimation over slots, i.e., $x(1), x(2), \ldots x(N),$ we have E-step given by

$$Q(u;u^{(k)}) = E(\sum_{n=1}^{N} \log f(z; u / x(n); u^{(k)})) \qquad (9)$$

the above equation can be further simplified as (after omitting constant terms)

$$Q(u;u^{(k)}) = \sum_{n=1}^{N} (-x(n)^H U\hat{\beta}(n) - \hat{\beta}(n)^H U^H x(n) + Trace(U^H U \hat{\beta}(n)\hat{\beta}(n)^H) \qquad (10)$$

where $\hat{\beta}(n) = E[\beta(n) / x(n); u^{(k)}],$ and $\hat{\beta}(n)\hat{\beta}(n)^H = E[\beta(n)\beta(n)^H / x(n); u^{(k)}]$

where E is the expectation operator. The conditional mean of fading coefficients will also be Gaussian. The conditional mean and second moment are easy to evaluate.

The maximization step (updated parameter values are given by the following equations.

$$\left.\frac{\partial Q}{\partial \tau}\right|_{\theta} = \frac{\partial Q}{\partial U}\left.(\frac{\partial U}{\partial \tau})\right|_{\theta} = 0 \qquad (11)$$

$$\left.\frac{\partial Q}{\partial \theta}\right|_{\tau} = \frac{\partial Q}{\partial U}\left.(\frac{\partial U}{\partial \theta})\right|_{\tau} = 0 \qquad (12)$$

Where

$$\frac{\partial Q}{\partial U} = \sum_{n=1}^{N} (-2\hat{\beta}(n)^H x(n) + 2\hat{\beta}(n)\hat{\beta}(n) U) \qquad (14)$$

These three equations can be solved iteratively to get the updated value of the parameters to be estimated. The above fixed point equations are nonlinear and hence there could be local minima or saddle points. That solution should be chosen which gives maximum value of the likelihood function. Detailed simulations results will be provided with the full paper.

References

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