

PARAMETERS IDENTIFICATION OF TIME-VARYING NON-GAUSSIAN AR MODEL USING WAVELETS

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Abstract: A new approach for parameters identification of time-varying non-Gaussian AR model is addressed in this paper. The proposed method is based on the application of higher-order spectra (HOS) and wavelet analysis. In order to solve the problem and identify the characteristics of the time-varying AR system, a time-varying parametric model is proposed as non-Gaussian AR model. The model parameters that characterize the time-varying system are functions of time and can be represented by a family of wavelet basis functions, of which the corresponding basis coefficients are invariant. This method enables us to closely track the changes in the model parameters. The simulation results also show the effectiveness of the proposed approach.

1. INTRODUCTION

Many of the existing methods for system identification and parameters estimation are exclusively based on the stationarity assumption. In spite of its many successful applications, aforementioned methods do not fulfill the nonstationary applications. Now more and more growing focus is put on the nonstationary environments, whose non-stationarity are close related with physiologic accommodation. The instantaneous information of those kinds of signals and systems are hard to be identified and predicted. There are many useful methods applied to nonstationary physical situations.

The most popular approach to estimate the nonstationary signals is to employ an adaptive algorithm. Some methods solved the problem by presuming that the process is locally stationary over a relatively short time interval but globally nonstationary. Then we look on this kind of signals as piece-stationary signals and the most important thing is to find the instants of change. More people construct the nonstationary models directly in terms of cumulant-based methods for time-varying AR model parameters estimation, but these methods were limited in certain AR parameters. A novel Bayesian formulation is developed to identify the system parameters and estimate the models. Other report uses a wavelet basis for the identification of time-varying system, and time-varying parameters can be expanded onto a finite set of wavelet basis sequences. Its flexibility in capturing the system's characteristics at different scales is at the cost of computational complexity. Several reports addressed the basis function. With the application of basis vectors such as Legendre polynomials and Fourier series, the time-varying model can be represented by a family of basis vectors, and the basis coefficients are invariant. We combine wavelet basis functions and the higher-order statistics, and propose a new method to estimate the coefficients of time-varying AR model, which is much better than the former methods.

2. THE PROPOSED MODEL

We build a time-varying time AR(p) model to extract the feature of the characteristics of the nonstationary signal. When $X = (X_1, X_2, \dots, X_n)$ is a nonstationary real process, the TV parameters of the model is employed, which is described by the following difference equation:

$$x(n) = -\sum_{k=1}^P a_k(n)x(n-k) + v(n) \quad (1)$$

where $v(k)$ is an independent identically distributed (i.i.d.) stationary, non-Gaussian process, with zero-mean and a finite nonzero cumulants. $a_k(n)$ are a time-varying parameter, which can be represented by a linear combination of a number of known functions:

$$a_k(n) = \sum_{j=0}^q a_{kj} u_j(n) \quad (2)$$

where $u_j(n)$ is the orthonormal basis functions. The time-varying AR coefficients are represented on the space spanned by the basis functions. time-varying AR coefficients are constants in this space. The functions are all known, so we can get $a_k(n)$ if we know the coefficients of the functions. The model can be described as the following:

$$x(n) = -\sum_{k=1}^P a_k(n)x(n-k) + v(n) = \sum_{k=1}^P \left[\sum_{j=0}^q a_{kj} u_j(n) \right] x(n-k) + v(n) \quad (3)$$

Some kinds of wavelet basis can be chosen to represent the AR coefficients, such as Harr and morlet basis because of their good performance in transient change.

3. RESULTS AND DISCUSSIONS

To test the performance of the proposed method, an AR model with order 2 is generated

$$x(n) = a_1(n)x(n-1) + a_2(n)x(n-2) + v(n)$$

where $v(k)$ is an independent identically distributed stationary, non-Gaussian process, whose variance $E[v^2(k)] = Q$ and $E[v^3(k)] = \beta \neq 0$. $\{a_1(k), a_2(k)\}$ are time-varying AR(2) model parameters changing abruptly in the following manner:

$$a_1(k) = -1.5, a_2(k) = 0.8, k \in [1, N/4] \cup [N/2 + 1, 3N/4],$$

$$a_1(k) = -0.9, a_2(k) = 0.2, k \in [N/4 + 1, N/2] \cup [3N/4 + 1, N],$$

The time-varying process is generated with $N=6000$ samples in Fig.1 (a) and (b), which are shown by the rectangle blue wave. Both Fourier basis and Harr basis are used to estimate the model parameters, and compare the results with the original values.

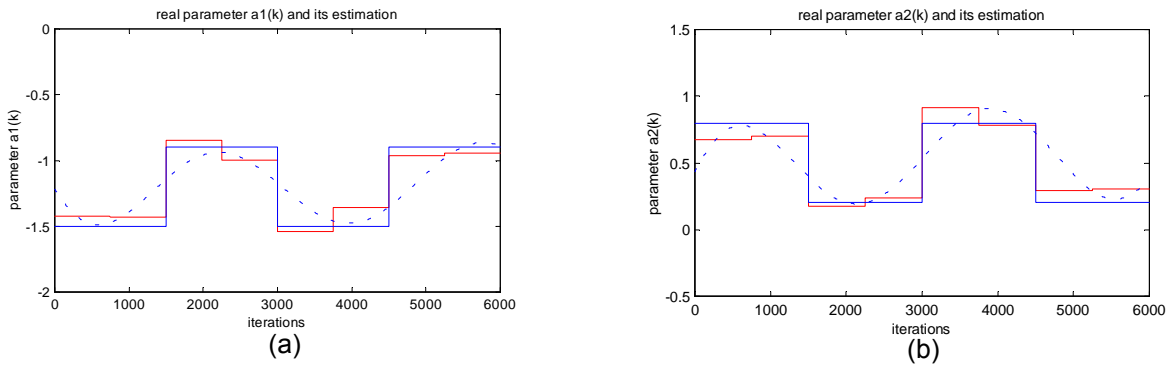


Fig. 1 (a) The estimated result of the time-varying $a_1(k)$. The estimated result of Fourier basis is shown as the dotted line, and the estimated result of Harr basis is shown as the solid line. (b) The result of the estimation of the time-varying $a_2(k)$. The estimated result based on Fourier basis is shown as the dotted line, and the estimated result based on Harr basis is shown as the solid line.

Both Fig.1 (a) and (b) depict the true AR(2) parameters $a_1(k)$ and $a_2(k)$, and the estimated results based on Fourier basis and Harr basis, respectively. We can see from the results that Harr basis is much better than Fourier basis and we can get the same result by the error criterion. The error is estimated by minimizing a penalized contrast function of the form

$$d_0(x_1, x_2, \dots, x_N; a_1, a_2 \dots a_p) = \sum_{i=P+1}^N [X_i - \sum_{j=1}^P \hat{a}_j \hat{X}_{i-j}]^2 / N \quad (4)$$

The error of wavelet basis with $d_{01} = 1.7332$ is less than the error of Fourier basis with $d_{02} = 2.0256$, which reflect that the wavelet basis is more suitable than the conventional method in the identification of the time-varying linear systems.

4. CONCLUSIONS

The aim of the proposed method is to investigate the problem of the parameters identification of the time-varying non-Gaussian AR model. The model parameters that characterize the time-varying system are functions of time and can be represented by a family of wavelet basis functions. A comparison between wavelet basis and Fourier basis of cumulants-based method is also demonstrated. The results in the presented method show the applicability and the effectiveness of the procedures. Some signal processing techniques is needed to apply to minimize the estimated error for the further work.