**Charles B. Coulbourn, et. al. "Area Measurement."**

# Area Measurement

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One must often measure the area of enclosed regions on plan-size drawings. These areas might be either regular or irregular in shape and describe one of the following:

- Areas enclosed by map contours
- Cross section of the diastolic and systolic volumes of heart cavities
- Farm or forest land shown in aerial photographs
- Cross sections of proposed and existing roads
- Quantities of materials used in clothing manufacture
- Scientific measurements
- Swimming pools
- Quantities of ground cover

Tools for this type of measurement include planimeters, digitizer-computer setups, digitizers with built-in area measuring capability, and grid overlay transparencies.

## 12.1 Theory

### Planimeter

A planimeter is a mechanical integrator that consists of a bar (tracer arm), a measuring wheel with its axis parallel to the bar, and a mechanism that constrains the movement of one end of the bar to a fixed track, Figure 12.1. The opposite end of the bar is equipped with a pointer for tracing the outline of an area. The measuring wheel, Figure 12.2, is calibrated with 1000 or more equal divisions per revolution. Each division equals one count. It accumulates counts, $P$, according to:

$$P = \frac{K}{\pi D} \int \sin \phi \, ds \tag{12.1}$$

where $K$ = number of counts per revolution of the measuring wheel
$D$ = diameter of the measuring wheel
$\phi$ = angle between the measuring wheel axis and the direction of travel
$s$ = traced path

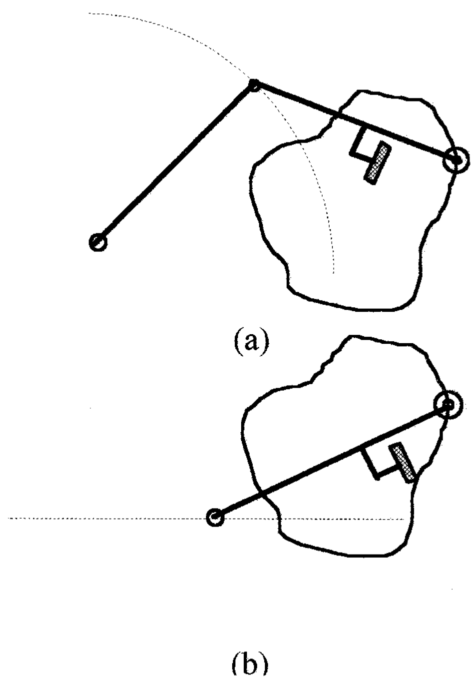


**FIGURE 12.1** The constrained end of a polar planimeter (*a*) follows a circular path; the constrained end of a linear planimeter (*b*) follows a straight line path.

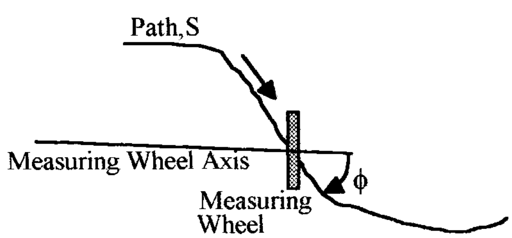


**FIGURE 12.2** The rotation of a measuring wheel is proportional to the product of distance moved and the sine of the angle between the wheel axis and direction of travel.

The size of an area, $A$, traced is:

$$A = \frac{P}{K} \times \pi D \times L \tag{12.2}$$

where $L$ = length of bar
$P$ = accumulated counts (Equation 12.1)

Figure 12.3 shows how a basic wheel and bar mechanism determines the area of a parallelogram. The traced path is along the sloped line; however, the wheel registers an amount that is a function of the product of the distance traveled and the sine of the angle between the direction of travel and the axis of the measuring wheel (Equation 12.1). This is the altitude of the parallelogram. The product of the altitude (wheel reading converted to distance) and base (bar length) is the area.

Figure 12.4(*a*) illustrates the operation of a planimeter when the area of a four-sided figure is measured. Figures 12.4(*b*), (*c*), (*d*), and (*e*) show the initial and final positions of the bar as each side of the figure

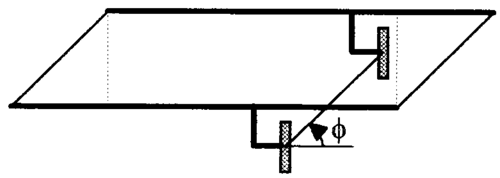


**FIGURE 12.3** The area of this parallelogram is proportional to the product of tracer arm length and measuring wheel revolutions.

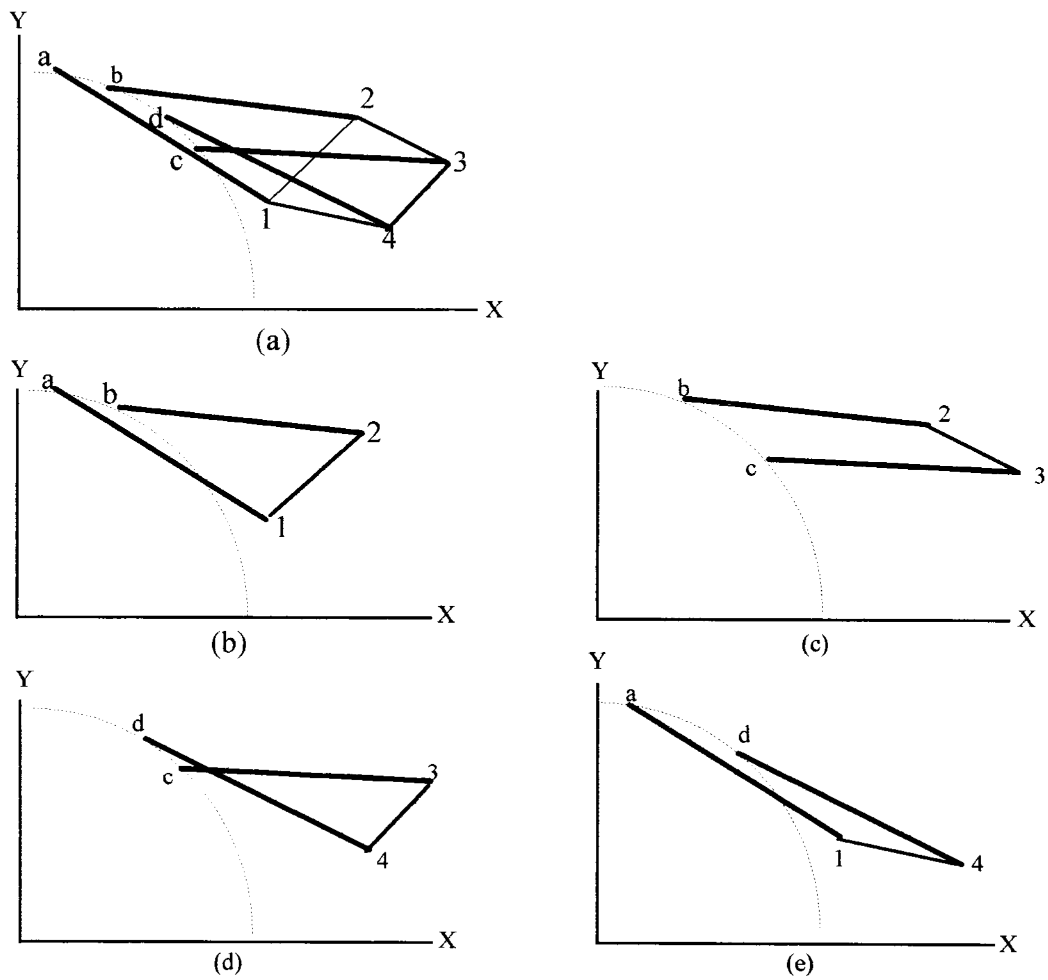


**FIGURE 12.4** This schematic shows a planimeter pointing to each junction of a four-sided figure being traced in (*a*) and at the ends of each of the segments in (*b*) through (*e*). The constrained end of the tracer arm follows a circle.

is traced. Applying the general expression for the area under a curve, $A = \int f(x)\,dx$ for each of these partial areas gives:

$$A_a = \left( \int_a^1 + \int_1^2 + \int_2^b + \int_b^a \right) f(x)\,dx \tag{12.3}$$

$$A_b = \left( \int_b^2 + \int_2^3 + \int_3^c + \int_c^b \right) f(x) dx \tag{12.4}$$

$$A_c = \left( \int_c^3 + \int_3^4 + \int_4^d + \int_d^c \right) f(x) dx \tag{12.5}$$

$$A_d = \left( \int_d^4 + \int_4^1 + \int_1^a + \int_a^d \right) f(x) dx \tag{12.6}$$

where $A_a$, $A_b$, $A_c$, and $A_d$ are the four partial areas.

The total area of the figure is the sum of the four partial areas. Combining the terms of these partial areas and rearranging them so that those defining the area traced by the left end of the bar are in one group, those defining the area traced by the other end of the bar are in a second group, and those remaining are in a third group results in the following:

$$\begin{aligned} A = \Bigg\{ & \left[ \int_b^a + \int_c^b + \int_d^c + \int_a^d \right] f(x) dx + \left[ \int_a^1 + \int_2^b + \int_b^2 + \int_3^c + \int_c^3 + \int_4^d + \int_d^4 + \int_1^a \right] f(x) dx \\ & + \left[ \int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \right] f(x) dx \Bigg\} \end{aligned} \tag{12.7}$$

The first four integrals describe the area traced by the left end of the bar. Since this end runs along an arc, it necessarily encloses an area equal to zero. The final four integral describe the area traced by the right end of the bar. This is the four-sided figure. The remaining eight integrals cancel out since $\int_a^1 + \int_1^a = 0$, etc. Thus, the total area equals the area traced by the right end of the bar. Note that the same reasoning applies to figures of any number of sides and of any shape.

## Digitizer

A digitizer converts a physical location on a map to digital code representing the $(x, y)$ coordinates of the location. The digital code is normally converted to a standard ASCII or binary format and transmitted to a computer where computations are made to determine such things as area or length. Certain digitizers can also compute areas and lengths without the use of a computer.

Area, $A$, can be computed using the coordinate pairs that define the area boundary.

$$\begin{aligned} A = & \frac{1}{2}(y_1 + y_2)(x_2 - x_1) + \frac{1}{2}(y_2 + y_3)(x_3 - x_2) + \ldots + \frac{1}{2}(y_{n-1} + y_n)(x_n - x_{n-1}) \\ & + \frac{1}{2}(y_n + y_1)(x_1 - x_n) \end{aligned} \tag{12.8}$$

where $x_1$, $x_2$, $x_3$, etc. = sequentially measured $x$ coordinates along the boundary.
$y_1$, $y_2$, $y_3$, etc. = are corresponding $y$ coordinates

All digitized coordinate pairs must be used in the computations since the $\Delta x$ intervals of this data are typically unequal.

In Equation 12.8, the final term deserves special consideration because it contains both the first and last coordinate pair of the series. When computations are performed from data stored in a computer, information is available for computing the final term so that no special actions are necessary. However, when the computations are performed by an embedded microprocessor with limited memory, normally each term is computed as the coordinates are read. In this case, one of two actions must be taken. First, the first coordinate pair is saved and then, once the measurement has been completed, a key is pressed to initiate computation of the final term. This is called "closing the area." Second, if the first coordinate pair is not saved, to prevent an error the final point digitized must coincide with the first point digitized so that $x_n = x_1$, $y_n = y_1$ and the final term is zero.

Popular types of digitizers in use today are tablets, sonic digitizers, and arm digitizers. Probably the most popular type is the tablet.

## Tablet Digitizer

Tablet digitizers consist of a pointer and a work surface containing embedded wires configured as a grid. The horizontal wires are parallel and spaced by about 12 mm. The vertical wires are also parallel and spaced the same. Different sensing techniques are used to locate the pointer position relative to the grid wires.

One sensing technique employs grid wires made from magnetostrictive material that has the property of changing shape very rapidly when subjected to a magnetic field. Each set of grid wires, the horizontal and vertical, is independently energized by a send wire that lies perpendicular to that set of wires [1]. A pulse transmitted over the send wire has a magnetic field that changes the shape of each magnetostrictive wire, causing a strain wave to propagate down the wire. Coincidentally, the pulse starts a counter. A coil in the pointer senses the strain wave and sends a signal to stop the counter. The counter reading is thus proportional to the propagation time of the strain wave. The product of the propagation time and velocity of the strain wave equals the physical distance between send line and pointer detector. The velocity of the strain waves is slow enough so that any errors in time measurement can be made acceptable.

A second sensing technique uses a grid made of conductive wires and a pointer that emits a signal on the order of 57.6 kHz [2]. The vertical grid wires are sensed to determine the amplitude distribution of signal induced in each wire. The point of maximum signal strength determines the location of the pointer along the horizontal axis. The horizontal grid wires are likewise sensed to find the point of maximum signal strength that determines the pointer location along the vertical axis. Coupling between the pointer and grid can be by either electromagnetic or by electrostatic means [3].

Signals can also be applied to the grid wires and the pointer used as a receiver. In this case, the signal to each grid wire must be coded or else applied sequentially, first to one set of grid wires and then to the other. Amplitude profiles of the signals received by the pointer are determined to locate the pointer along each axis.

## Sonic Digitizers

Sonic digitizers consist of a pointer and two or more microphones. The pointer, used to identify the point to be digitized, typically contains a spark gap that periodically emits a pulse of sonic energy [4]. The microphones in certain cases are mounted in a bar that is located along the top of the drawing area. In other cases, they are mounted in an L frame and located along the top and one side of the drawing area.

The microphones receive the sonic pulse emitted by the pointer. The time taken for the pulse to travel from the transmitter to each receiver is measured and the slant distance computed from the product of this elapsed time and the sonic velocity. The $x$ and $y$ coordinates of the pointer are computed from the slant ranges and the locations of the microphones.

Ambiguities exist since a single set of slant ranges describe two points; however, the ambiguities can be resolved either by using additional microphones or by ensuring that the ambiguous points are outside the work area. With the microphones aligned along the top of the work area, the ambiguous points are

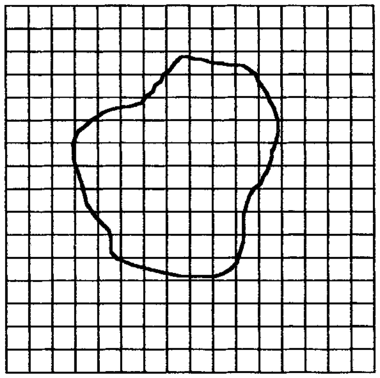

**FIGURE 12.5** Grid overlays offer a simple and readily available method for measuring areas.

outside the work area; with them aligned along top and side, the ambiguities have to be resolved with additional microphones.

The sound wave velocity is slow enough so that errors in time measurements can be made acceptable.

### Arm Digitizer

An arm digitizer consists of a base, two arms, two rotary encoders, and a pointer. The *base* must be anchored at a point removed from the work area, normally at the top of the user's desk. One end of the arm, called the *base arm*, swivels about a vertical axis at the base. The other end of the base arm hinges to one end of a second arm, called the *tracer arm*, about a vertical axis. On the other end of the tracer arm is the *pointer*. One of the encoders, the *base encoder*, detects the angle between the base and the base arm. The other encoder, the *arm encoder*, detects the angle between the base arm and the tracer arm.

To simplify implementation, each arm is made the same length and, for sufficient accuracy, each encoder provides in excess of 36,000 counts per revolution. The encoder output signals consist of dual square waves with a 90° phase relationship. The leading and trailing edges of each square wave are counted and the 90° phase relationship provides count direction.

Since each encoder is a relative rather than absolute counting device, the count registers of each encoder must be initialized at a known angle. The count register of the arm encoder is set to the angle that results when the pointer is moved to a precisely known location. This location is called "home." The count register for the base encoder is initialized when the rotation of the $x$–$y$ coordinate system is set. Thereafter, the encoders add to or subtract from their count registers as the pointer is moved.

With the arm lengths and the angles precisely known, the $x$, $y$ position of the pointer can be computed. However, $x$ and $y$ are relative to a coordinate system whose origin is at the axis of the base and whose rotation is unknown. The user must therefore select an origin, usually near the lower left corner of the drawing area, and the direction of the $x$-axis, usually parallel to the bottom edge of the drawing area. This information is sufficient for the processor to compute the correct translation and rotation or the coordinate system.

### Grid Overlay

A grid overlay is simply a transparent sheet onto which a grid has been drawn. To use it, place it atop the drawing or photo of the area to be measured (Figure 12.5). Then count the number of squares that lie within the boundary of the area. Squares that are at least half enclosed should be counted. The unknown area equals the product of the number of blocks counted and the area of each block. The accuracy achieved is dependent on grid size, precision of the grid dimensions, and counting accuracy.

## 12.2 Equipment and Experiment

Many different types of planimeters and digitizers are manufactured in the U.S., Europe, and Japan. A representative sample of these instruments are described in this section.

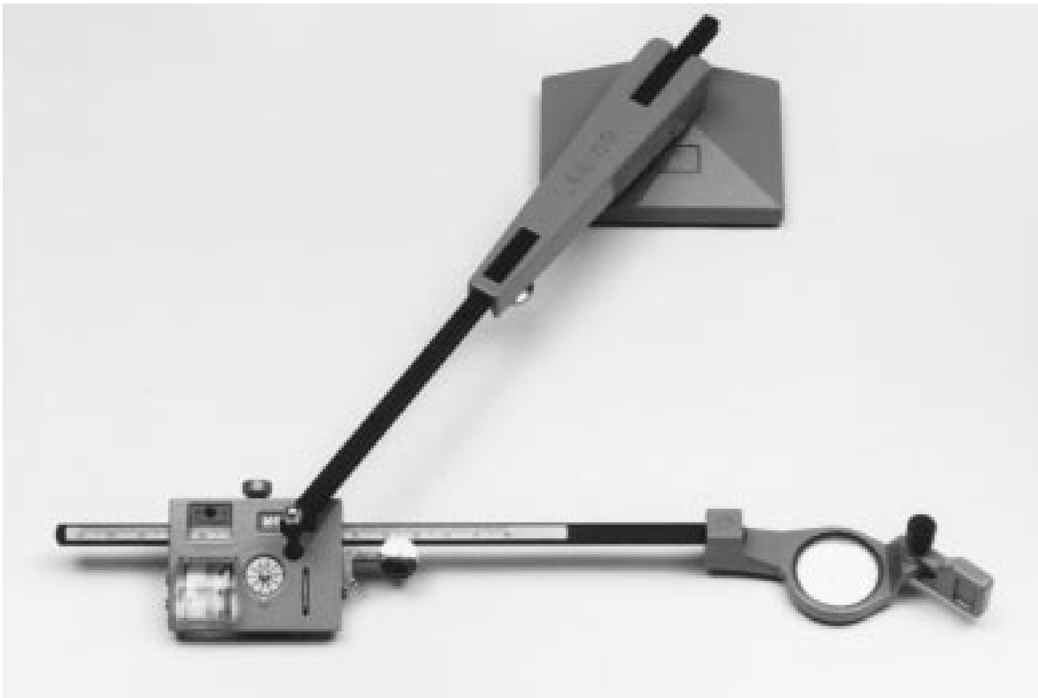

**FIGURE 12.6** Mechanical planimeters are normally preferred when occasional use is required.

One of the simplest and least costly area measuring devices is the manual polar planimeter (Figure 12.6). It consists of a weight, two arms, a measuring wheel, and a pointer. The *weight* secures one end of a *pole arm*, allowing the other end to rotate along a fixed arc. The rotating end of the pole arm attaches to one end of a *tracer arm* and constrains its movement to the arc. At the other end of the tracer arm is a *pointer* used for tracing the periphery of an unknown area. The *measuring wheel* is located in the box at one end of the tracer arm. The location of the measuring wheel is not critical; however, its axle must be parallel to the tracer arm.

The length of both arms of the planimeter shown in Figure 12.6 can be adjusted. The length of the pole arm has absolutely no effect on measurement accuracy and is adjustable only for convenience. The effective length of the tracer arm directly affects the reading; a shorter arm results in a larger reading. By adjusting the tracer arm length, one can achieve a very limited range of scaling; however, this is usually not done. Rather, the arm length is adjusted according to the general size of areas to be measured: a shorter arm for smaller areas and a longer arm for larger areas. A shorter arm results in a greater number of counts per unit area, which is needed for smaller areas. Scaling is usually done by multiplying the result by an appropriate value.

One zeros the measuring wheel of the planimeter shown in Figure 12.6 by turning a small knurled wheel attached to the measuring wheel axle. On other models, one pushes a plunger to zero the wheel. The latter method is easier but is sensitive to misalignment.

A planimeter pointer is usually a lens with a small circle engraved in the center, although some planimeter models use a needle as the pointer. The best type of pointer is a matter of personal preference although lens pointers are much more popular.

Figure 12.7 shows two electronic planimeters with digital readouts. With these planimeters, one can measure length as well as area. To measure length, snap out the measuring wheel housing, attach an auxiliary handle, and roll the wheel along the line to be measured. Extremely high accuracy can be achieved.

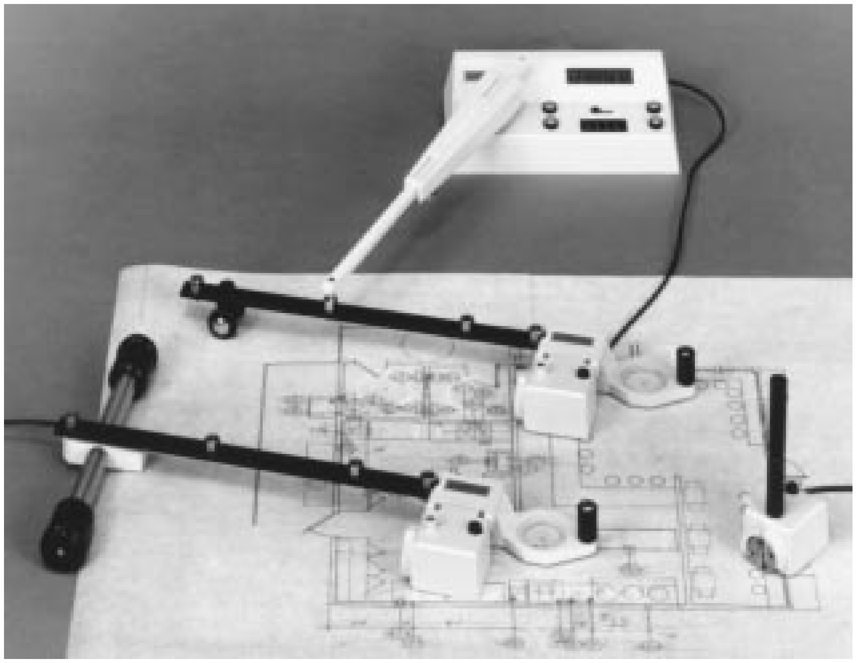

**FIGURE 12.7** Electronic planimeters are easier to use and read, and are preferred especially when frequently used.

The upper planimeter in Figure 12.7 is a polar type. It consists of the same parts as the model in Figure 12.6, except that the measuring wheel is attached to a small optical encoder and the weight is packaged in the digital readout. The encoder provides two square-wave outputs that are in phase quadrature, that is, one is 90° out of phase with the other. Both outputs are fed to a processor that counts the pulses and uses the phase difference to determine count direction.

The processor has an electronic scale feature that translates the planimeter reading to real measurement units such as square feet, square meters, acres, or hectares. One can transmit processor data to a computer using an auxiliary interface unit (not shown).

The planimeter at the bottom of Figure 12.7 is a linear model since the path traveled by the constrained end of the tracer arm is a straight line. The straight line path is maintained by a rigid connection between the two carriage wheels and their axle. Other linear planimeters use an actual rail to guide the constrained end of the tracer arm along a straight line path.

The polar planimeter shown at the top of Figure 12.8 and the linear planimeter shown at the bottom of Figure 12.8 are both compact battery-operated models. The measuring wheel is built into the processor, which is attached to the pole arm of the planimeter. The effective length of the tracer arm for both instruments extends from the axis of the constrained end to the pointer, which for these instruments is a small circle engraved in the center of a lens. These planimeters provide electronic scaling and averaging of multiple readings. They cannot be used to measure length.

Figure 12.9 shows an arm digitizer that can be used either as a stand-alone area and length measuring device or to digitize a map or drawing. When operating as a digitizer, the arm digitizer displays the $(x, y)$ coordinates as well as transmits them to a computer. The digitizer has a built-in interface and can transmit using any of over 24 different ASCII and binary codes, each with a choice of parameters. It can be set to measure $x$ and $y$ coordinates in either English or metric units.

Three other modes, in addition to the digitizer mode, are available for computing and displaying either area and length, area and item count, or item count and length. Measurements can also be made in either English or metric units. Any displayed item can be transmitted to a computer through the built-in interface.

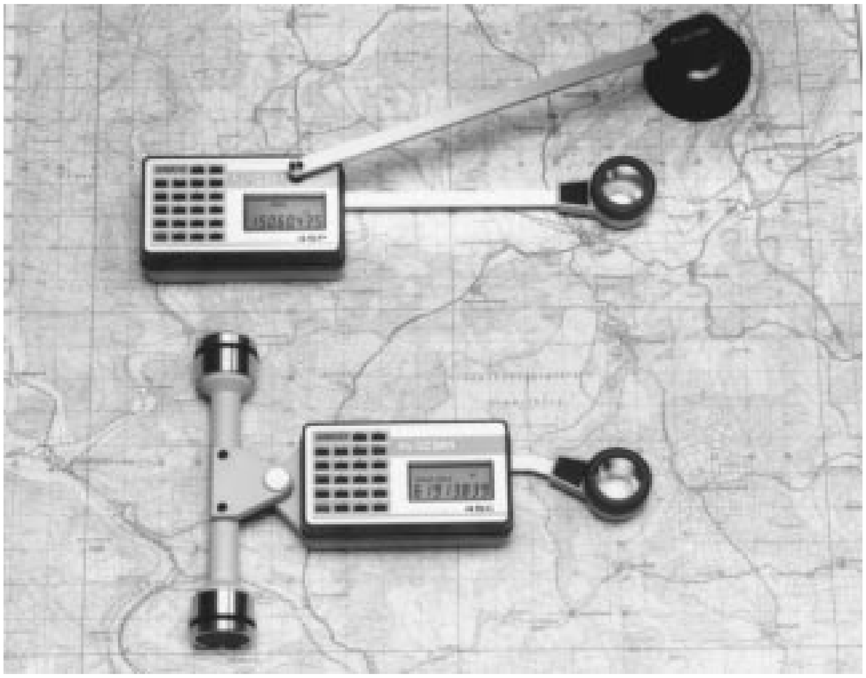

**FIGURE 12.8** Battery-operated planimeters have the advantages of electronics planimeters and are portable.

**FIGURE 12.9** An arm digitizer lets one measure area, length, and coordinates, and transmit the displayed data to a computer; it requires minimum desk space.

**FIGURE 12.10** The popular pad digitizer includes a lightweight puck or pen-type pointer for selecting points whose coordinates are transmitted to a computer for processing.

The arm digitizer has other features that are especially useful when it is used in a nondigitizer mode. It provides independent scaling in each axis, empirical scale factor determination, key-press programming, four-function calculating, unit conversion, and averaging. Also, the arm digitizer can be folded out of the way when not in use.

Arm digitizers can be used on transparent surfaces as well as on normal opaque work surfaces. Thus, they are also useful for digitizing or measuring areas on rear-projected maps or drawings.

Tablet digitizers, like the one shown in Figure 12.10, are popular instruments used for digitizing maps and drawings and as extended keypads for certain software. When used as an extended keypad, a template with figures of key functions is placed somewhere in the work area. Pointing to a function and clicking a cursor key selects it. Tablets feature a built-in interface and one or more binary and ASCII codes. Communication parameters on some tablets are controlled by dip switches and on others by computer software. Pointers are lightweight and include between 1 and 16 cursor keys. Tablets are available in sizes ranging from that of a notebook up to 122 cm by 168 cm (48 in. by 66 in.) or more. Models are available with transparent work surfaces and can be used with rear-projected maps and drawings.

Tablet digitizers have the advantage of lightweight pointers but the larger tablets have the disadvantages of occupying a significant amount of floor space and being relatively difficult to move around. One manufacturer has overcome this disadvantage by designing a digitizer tablet that rolls up.

Sonic digitizers, like tablets, are intended for digitizing maps and drawings and for providing an extended keypad for certain software packages. They feature a built-in computer interface with a number of ASCII and binary codes, with a choice of parameters, that make them suitable for use with many different software packages. A major advantage enjoyed by sonic digitizers is their portability and the fact that they operate well with a transparent work surface required for rear-projected drawings and maps.

Table 12.1 provides a list of manufacturers of planimeters and Table 12.2 lists many of the manufacturers of graphic digitizers.

**TABLE 12.1** Companies that Make Planimeters

| | |
|---|---|
| Gebruder Haff Gmbh<br>Tiroler Strasse 5<br>D-87459 Pfronten<br>Germany<br>Tel: 49-8363-9122-0 | Lasico Inc.<br>2451 Riverside Drive<br>Los Angeles, CA 90039<br>Tel: (213) 662-2128 |
| Koizumi Sokki Mfg. Co., Ltd.<br>1-132, Midori-Cho, Nagaoka-Shi<br>Niigata 940-21<br>Japan<br>Tel: (0) 258-27-1102 | Sokkia Corp.<br>9111 Barton St.<br>P.O. Box 2934<br>Overland Park, KS<br>Tel: (800) 476-5542 |

**TABLE 12.2** Companies that Make Digitizers

| | |
|---|---|
| Altek Corporation<br>12210 Plum Orchard St.<br>Silver Spring, MD 20904<br>Tel: (301) 572-2555 | Lasico Inc.<br>2451 Riverside Drive<br>Los Angeles, CA 90039<br>Tel: (213) 662-2128 |
| Calcomp Technology Inc.<br>2411 West La Palma Ave.<br>Anaheim, CA 92801-2589<br>Tel: (800) 445-6515 | Numonics Corporation<br>101 Commerce Drive<br>Box 1005<br>Montgomeryville, PA 18936<br>Tel: (215) 362-2766 |
| GTCO Corporation<br>7125 Riverwood Drive<br>Columbia, MD 21046<br>Tel: (800) 344-4723 | Wacom Technology Corp.<br>501 S.E. Columbia Shores Blvd., Suite 300<br>Vancouver, WA 98661<br>Tel: (360) 750-8882 |

## 12.3 Evaluation

Each of the area measuring devices described in this section are excellent and have been thoroughly proven by use. However, some of the devices are more suited to certain tasks and operating environments than others.

To measure an area that is smaller than an equivalent circle of about 2 cm in diameter, a digitizer is probably the best choice. For areas of this magnitude, the resolution element of planimeters starts to become a significant portion of the total area. The resolution element of most digitizers is significantly less than that of a planimeter, and any measurement is always plus- or -minus a resolution element.

For measuring areas that are equivalent to a circle between 2 cm and 55 cm in diameter, either a planimeter or a digitizer will provide excellent results.

When measuring areas larger than an equivalent circle of about 55 cm in diameter, one can still use a planimeter; however, one must subdivide the large area into smaller areas and then individually measure each of the smaller areas. A digitizer can measure significantly larger areas, but at some point it too will reach a limit. Then, one can use the same technique of subdividing the large area into smaller areas that the digitizer can handle.

Area measuring instruments of the future will undoubtedly make even greater use of microprocessors to provide more features such as incorporation of slope and tilt correction, statistical operations, and determination of centroids, moment, etc. The instruments should become mechanically simpler and more reliable. Features such as conservation of office space and portability will be emphasized. The ultimate area measuring device will consist of a detached cursor for pointing and a small calculator-like device for operating on the results, displaying them, storing them, and sending them to a computer.

Scanners and associated software will also impact the field of area measurement, particularly as their coverage increases and their price decreases.

## Defining Terms

**Planimeter:** A mechanical integrator used for measuring the area of enclosed regions on maps, plans, etc.

**Pole arm:** One of the two bars comprising a polar planimeter. One end of the pole arm is fixed and the other end is free to rotate. The length of the pole arm has no effect on the planimeter reading.

**Tracer arm:** The bar of a planimeter to which is attached the measuring wheel. One end of the tracer arm is constrained to a fixed path, while the other end traces the perimeter of an enclosed region whose area is being measured. The length of the pole arm is indirectly proportional to the planimeter reading.

**Polar planimeter:** A planimeter with a tracer arm whose constrained end follows a circle.

**Linear planimeter:** A planimeter with a tracer arm whose constrained end follows a straight line.

**Measuring wheel:** The planimeter wheel whose degree of rotation is directly proportional to area.

**Digitizer:** A device to convert data or an image to digital form. The digitizers discussed here convert images to digital form and are categorized as graphic digitizers.

**Pointer:** The part of a planimeter or digitizer that is used to follow the line being traced.

**Resolution element:** The smallest elemental area that can be discerned. When referred to in connection with area measurement, it is an area with a value of 1.

## References

1. P. A. Santi, J. Fryhofer, and G. Hansen, Electronic planimetry, *Byte*, March 1980, 113–122.
2. K. Mandelberg, Anonymous, *Internet*, 4–96.
3. E. Jones, ’89 Planning guide-digitizers, *Architectural & Engineering*, July 1980, 37–40.
4. P. E. Maybaum, Digitizing and computer graphics, *Keyboard*, Sept. 1978, 1–3.

## Further Information

F. A. Willers, *Mathematische Instrumente*, Munchen und Berlin: Verlag von R. Oldenbourg, 1943.