Halit Eren. "Acceleration, Vibration, and Shock Measurement."

Acceleration, Vibration, and Shock Measurement

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Acceleration is measured by accelerometers as an important parameter for general-purpose absolute motion measurements, and vibration and shock sensing. Accelerometers are commercially available in a wide variety of ranges and types to meet diverse application requirements. They are manufactured small in size, light in weight, rugged, and robust to operate in harsh environment. They can be configured as active or passive sensors. An active accelerometer (e.g., piezoelectric) gives an output without the need for an external power supply, while a passive accelerometer only changes its electric properties (e.g., capacitance) and requires an external electrical power. In applications, the choice of active or passive type accelerometer is important, since active sensors cannot measure static or dc mode operations. For

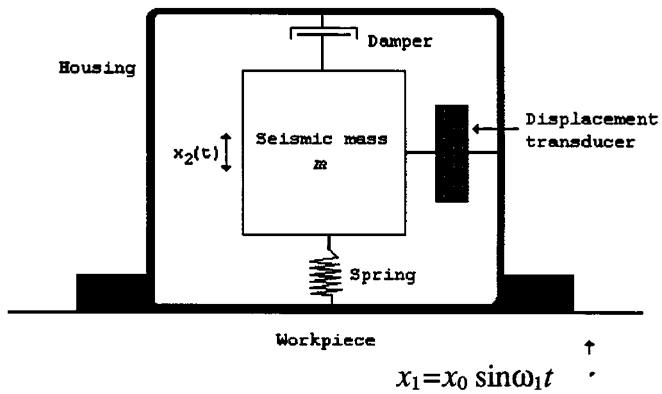

FIGURE 17.1 A typical deflection-type seismic accelerometer. In this basic accelerometer, the seismic mass is suspended by a spring or cantilever inside a rigid frame. The frame is connected to the vibrating structure; as vibrations take place, the mass tends to remain fixed so that relative displacements can be picked up. They are manufactured in many different types and sizes and they exhibit diverse characteristics.

true static measurements, passive sensors must be used. In general, accelerometers are preferred over displacement and velocity sensors for the following reasons:

1. They have a wide frequency range from zero to very high values. Steady accelerations can easily be measured.
2. Acceleration is more frequently needed since destructive forces are often related to acceleration rather than to velocity or displacement.
3. Measurement of transients and shocks can readily be made, more easily than displacement or velocity sensing.
4. Displacement and velocity can be obtained by simple integration of acceleration by electronic circuitry. Integration is preferred over differentiation.

Accelerometers can be classified in a number of ways, such as *deflection* or *null-balance* types, mechanical or electrical types, dynamic or kinematic types. The majority of industrial accelerometers can be classified as either deflection type or null-balance type. Those used in vibration and shock measurements are usually the deflection types, whereas those used for measurements of motions of vehicles, aircraft, etc. for navigation purposes may be either type. In general, null-balance types are used when extreme accuracy is needed.

A large number of practical accelerometers are of the deflection type; the general configuration is shown in Figure 17.1. There are many different deflection-type accelerometers. Although their principles of operation are similar, they only differ in minor details, such as the spring elements used, types of damping provided, and types of relative motion transducers employed. These types of accelerometers behave as second-order systems; the detailed mathematical analysis will be given in later sections.

Accelerometers can be classified as *dynamic*, meaning that the operation is based on measuring the force required to constrain a seismic mass to track the motion of the accelerated base, such as spring-constrained-slug types. Another type is the *kinematic* accelerometer, which is based on timing the passage of an unconstrained proof mass from spaced points marked on the accelerated base; this type is found in highly specific applications such as interspace spacecraft and in gravimetry type measurements.

For practical purposes, accelerometers can also be classified as *mechanical* or *electrical*, depending on whether the restoring force or other measuring mechanism is based on mechanical properties, (e.g., the law of motion, distortion of a spring or fluid dynamics, etc.) or on electrical or magnetic forces.

Calibrations of accelerometers are necessary in acceleration, vibration, and shock sensing. The calibration methods can broadly be classified to be *static* or *dynamic*. Static calibration is conducted at one or several levels of constant acceleration. For example, if a tilting table calibration method is selected, the vertical component of the free fall is used without a choice of magnitude. On the other hand, if a centrifuge is selected, it produces a constant acceleration as a function of the speed of rotation, and the magnitudes can be chosen in a wide range from 0 to well over 50,000 g. The dynamic calibration is usually done using an electrodynamic shaker. The electrodynamic shaker is designed to oscillate in a sinusoidal motion with variable frequencies and amplitudes. They are stabilized at selected levels of calibration. This is an absolute method that consists of measuring the displacement with a laser interferometer and a precise frequency meter for accurate frequency measurements. The shaker must be driven by a power amplifier, thus giving a sinusoidal output with minimal distortion. The National Bureau of Standards uses this method as a reference standard. Precision accelerometers, mostly of the piezoelectric type, are calibrated by the absolute method and then used as the working standard. A preferred method is back-to-back calibration, where the test specimen is directly mounted on the working standard that, in turn, is mounted on an electrodynamic shaker.

Before providing details of different type of accelerometers, the common features such as accelerometer dynamics, velocity, distance, shock frequency responses, etc. will be introduced in the next section.

17.1 Accelerometer Dynamics: Frequency Response, Damping, Damping Ratio, and Linearity

This section concerns the physical properties of acceleration, vibration, and shock measurements in which accelerometers are commonly used. A full understanding of accelerometer dynamics is necessary in relation to characteristics of acceleration, vibration, and shock. The vibrations can be periodic, stationary random, nonstationary random, or transient.

Periodic Vibrations

In periodic vibrations, the motion of an object repeats itself in an oscillatory manner. This can be represented by a sinusoidal waveform:

$$x(t) = X_{peak} \sin(\omega t) \tag{17.1}$$

where $x(t)$ = time-dependent displacement
ω = $2\pi f t$ = angular frequency
X_{peak} = maximum displacement from a reference point

The velocity of the object is the time rate of change of displacement:

$$u(t) = \mathrm{d}x/\mathrm{d}t = \omega X_{peak} \cos(\omega t) = U_{peak} \sin(\omega t + \pi/2) \tag{17.2}$$

where $u(t)$ = time-dependent velocity
U_{peak} = ωX_{peak} = maximum velocity

The acceleration of the object is the time rate change of velocity:

$$a(t) = \mathrm{d}u/\mathrm{d}t = \mathrm{d}^2 u/\mathrm{d}t^2 = -\omega^2 X_{peak} \sin(\omega t) = A_{peak} \sin(\omega t + \pi) \tag{17.3}$$

where $a(t)$ = time-dependent acceleration
A_{peak} = $\omega^2 X_{peak} = \omega\ U_{peak}$ = maximum acceleration

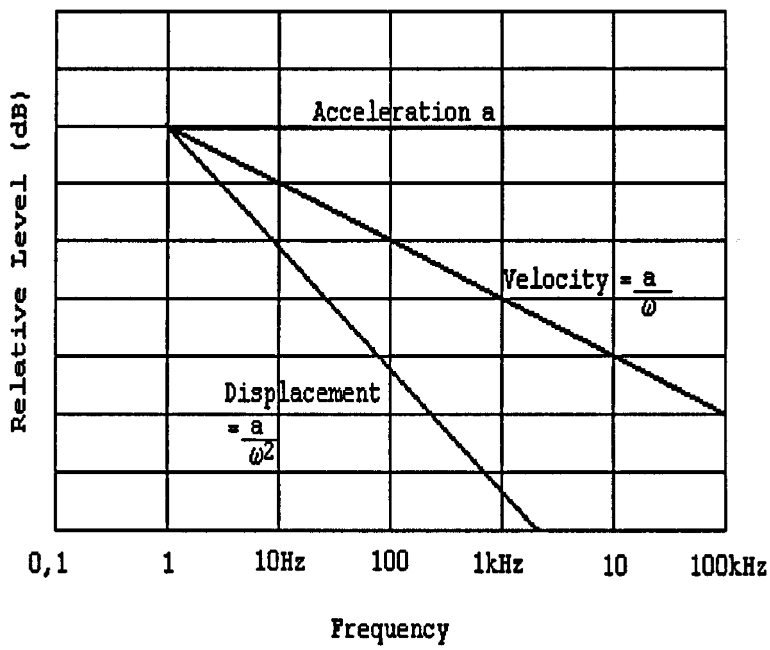

FIGURE 17.2 Logarithmic relationship between acceleration, velocity, and displacement. Velocity at a particular frequency can be obtained by dividing acceleration by a factor proportional to frequency. For displacement, acceleration must be divided by a factor proportional to the square of the frequency. Phase angles need to be determined separately, but they can be neglected in time-averaged measurements.

From the above equations, it can be seen that the basic form and the period of vibration remains the same in acceleration, velocity, and displacement. But velocity leads displacement by a phase angle of 90° and acceleration leads velocity by another 90°. The amplitudes of the three quantities are related as a function of frequency, as shown in Figure 17.2.

In nature, vibrations can be periodic but not necessarily sinusoidal. If they are periodic but nonsinusoidal, they can be expressed as a combination of a number of pure sinusoidal curves, described by Fourier analysis as:

$$x(t) = X_0 + X_1 \sin(\omega_1 t + \phi_1) + X_2 \sin(\omega_2 t + \phi_2) + \ldots X_n \sin(\omega_n t + \phi_n) \qquad (17.4)$$

where $\omega_1, \omega_2, \ldots, \omega_n$ = frequencies (rad s^{-1})
$X_0, X_1, \ldots, X_n$ = maximum amplitudes of respective frequencies
$\phi_1, \phi_2, \ldots, \phi_n$ = phase angles

The number of terms may be infinite: the higher the number of elements, the better the approximation. These elements constitute the *frequency spectrum.* The vibrations can be represented in time domain or frequency domain, both of which are extremely useful in the analysis. As an example, in Figure 17.3, the time response of the seismic mass of an accelerometer is given against a rectangular pattern of excitation of the base.

Stationary Random Vibrations

Random vibrations are often met in nature where they constitute irregular cycles of motion that never repeat themselves exactly. Theoretically, an infinitely long time record is necessary to obtain a complete description of these vibrations. However, statistical methods and probability theory can be used for the

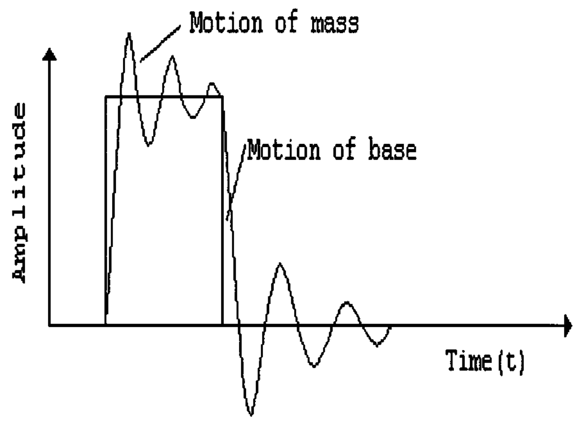

FIGURE 17.3 Time response of a shock excitation of a single degree-of-freedom system. As the duration of the shock pulse increases, sustained oscillations get shorter in time but larger in amplitude. The maximum system response may be as high as twice the magnitude of the shock pulse.

analysis by taking representative samples. Mathematical tools such as probability distributions, probability densities, frequency spectra, cross- and auto-correlations, Digital Fourier Transforms (DFT), Fast Fourier Transforms (FFT), auto spectral analysis, RMS values, and digital filter analysis are some of the techniques that can be employed. Interested readers should refer to references for further information.

Transients and Shocks

Often, short-duration and sudden-occurrence vibrations need to be measured. Shock and transient vibrations may be described in terms of force, acceleration, velocity, or displacement. As in the case of random transients and shocks, statistical methods and Fourier Transforms are used in the analysis.

Nonstationary Random Vibrations

In this case, the statistical properties of vibrations vary in time. Methods such as time averaging and other statistical techniques can be employed. A majority of accelerometers described here can be viewed and analyzed as seismic instruments consisting of a mass, a spring, and a damper arrangement, as shown in Figure 17.1. Taking only the mass-spring system, if the system behaves linearly in a time invariant manner, the basic second-order differential equation for the motion of the mass alone under the influence of a force can be written as:

$$f(t) = m\mathrm{d}^2x/\mathrm{d}t^2 + c\,\mathrm{d}x/\mathrm{d}t + kx \tag{17.5}$$

where $f(t)$ = force
m = mass
c = velocity constant
k = spring constant

Nevertheless, in seismic accelerometers, the base of the arrangement is also in motion. Therefore, Equation 17.5 can be generalized by taking the effect motion of the base into account. Then, Equation 17.5 can be modified as:

$$m\mathrm{d}^2z/\mathrm{d}t^2 + c\,\mathrm{d}z/\mathrm{d}t + kz = mg\cos(\theta) - m\mathrm{d}^2x_1/\mathrm{d}t^2 \tag{17.6}$$

where $z = x_2 - x_1$ = the relative motion between the mass and the base
x_1 = displacement of the base
x_2 = displacement of the mass
θ = the angle between sense axis and gravity

In order to lay a background for further analysis, taking the simple case, the complete solution to Equation 17.5 can be obtained by applying the superposition principle. The superposition principle states that if there are simultaneously superimposed actions on a body, the total effect can be obtained by summing the effects of each individual action.

Equation 17.5 describes essentially a second-order system that can be expressed in Laplace transform as:

$$X(s)/F(s) = 1/ms^2 + cs + k \tag{17.7}$$

or

$$X(s)/F(s) = K/\left[s^2/\omega_n^{\ 2} + 2\zeta s/\omega_n + 1\right] \tag{17.8}$$

where s = the Laplace operator
K = $1/k$ = static sensitivity
ω_n = $\sqrt{k/m}$ = undamped critical frequency, rad/s
ζ = $c/2\sqrt{km}$ = damping ratio

As can be seen, in the performance of accelerometers, important parameters are the static sensitivity, the natural frequency, and the damping ratio, which are functions of mass, velocity, and spring constants. Accelerometers are designed to have different characteristics by suitable selection of these parameters.

Once the response is expressed in the form of Equations 17.7 and 17.8, analysis can be taken further, either in the time domain or in the frequency domain. The time response of a typical second-order system for a unit-step input is given in Figure 17.4. The Bode plot gain phase responses are depicted in Figure 17.5. Detailed discussions about frequency response, damping, damping ratio, and linearity are made in relevant sections, and further information can be obtained in the references.

Systems in which a single structure moves in more than one direction are termed *multi-degree-of-freedom systems*. In this case, the accelerations become functions of dimensions as d^2x/dt^2, d^2y/dt^2, and d^2z/dt^2. Hence, in multichannel vibration tests, multiple transducers must be used to create uniaxial, biaxial, or triaxial sensing points for measurements. Mathematically, a linear multidegree-of-freedom system can be described by a set of coupled second-order linear differential equations; and when the frequency response is plotted, it normally shows one resonance peak per degree of freedom.

Frequently, acceleration and vibration measurements of thin plates or small masses are required. Attaching an accelerometer with a comparable mass onto a thin plate or a small test piece can cause "mass loading." Since acceleration is dependent on the mass, the vibration characteristics of the loaded test piece could be altered, thus yielding wrong measurements. In such cases, a correct interpretation of the results of the measuring instruments must be made. Some experimental techniques are also available for the correction of the test results in the form of performing repetitive tests conducted by sequentially adding small known masses and by observing the differences.

The following sections discuss different types of accelerometers.

17.2 Electromechanical Force-Balance (Servo) Accelerometers

Electromechanical accelerometers, essentially servo or null-balance types, rely on the principle of feedback. In these instruments, acceleration-sensitive mass is kept very close to a neutral position or zero displacement point by sensing the displacement and feeding back this displacement. A proportional

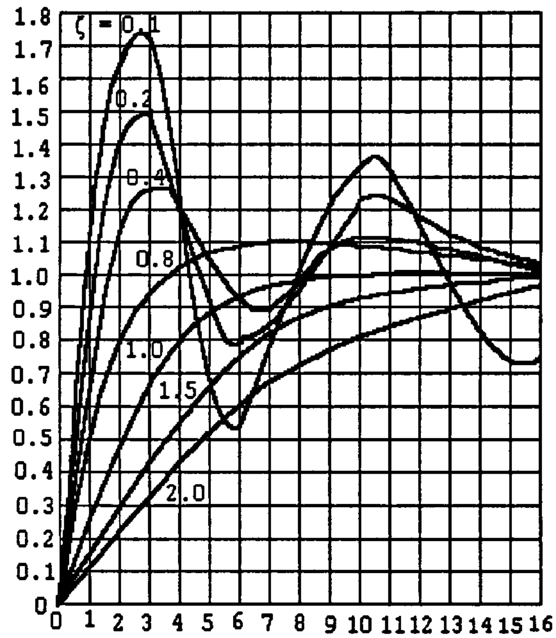

FIGURE 17.4 Unit step time responses of a second-order system with various damping ratios. The maximum overshoot, delay, rise, settling times, and frequency of oscillations depend on the damping ratio. Smaller damping ratios give faster response but larger overshoot. In many applications, a damping ratio of 0.707 is preferred.

magnetic force is generated to oppose the motion of the mass displaced from the neutral, thus restoring neutral position — just as a mechanical spring in a conventional accelerometer would do. The advantages of this approach are the better linearity and elimination of hysteresis effects as compared to mechanical springs. Also, in some cases, electric damping can be provided, which is much less sensitive to temperature variations.

One very important feature of null-balance type instruments is the capability of testing the static and dynamic performances of the devices by introducing electrically excited test forces into the system. This remote self-checking feature can be quite convenient in complex and expensive tests where it is extremely critical that the system operates correctly before the test commences. They are also useful in acceleration control systems, since the reference value of acceleration can be introduced by means of a proportional current from an external source. They are usually used for general-purpose motion measurements and monitoring low-frequency vibrations. They are specifically applied in measurements requiring better accuracy than achieved by those accelerometers based on mechanical springs as the force-to-displacement transducer.

There are a number of different types of electromechanical accelerometers: coil-and-magnetic types, induction types, etc.

Coil-and-Magnetic Type Accelerometers

These accelerometers are based on Ampere's law; that is: "a current carrying conductor disposed within a magnetic field experiences a force proportional to the current, the length of the conductor within the field, the magnetic field density, and the sine of the angle between the conductor and the field."

Figure 17.6 illustrates one form of accelerometer making use of the above principle. The coil is located within the cylindrical gap defined by a permanent magnet and a cylindrical soft iron flux return path. It is mounted by means of an arm situated on a minimum friction bearing so as to constitute an acceleration-sensitive seismic mass. A pick-off mechanism senses the displacement of the coil under

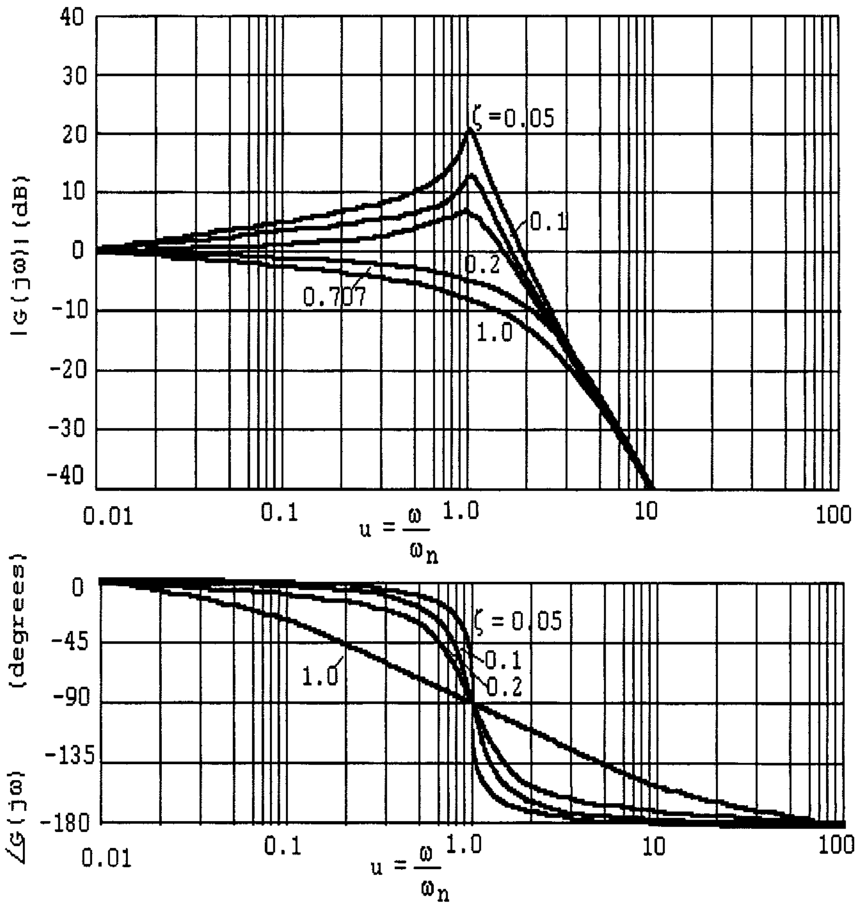

FIGURE 17.5 Bode plots of gains and phase angles against frequency of a second-order system. Curves are functions of frequencies as well as damping ratios. These plots can be obtained theoretically or by practical tests conducted in the frequency range.

acceleration and causes the coil to be supplied with a direct current via a suitable servo-controller to restore or maintain a null condition.

Assuming a downward acceleration with the field being radial (90°) and using Ampere's law, the force experienced by the coil may be written as:

$$F = ma = ilB \tag{17.9}$$

or the current

$$i = ma/lB \tag{17.10}$$

where B = the effective flux density
l = the total effective length of the conductor in the magnetic field

Current in the restoring circuit is linearly proportional to acceleration, provided: (1) armature reaction effects are negligible and fully neutralized by the compensating coil in opposition to the moving coil, and (2) the gain of the servo system is large enough to prevent displacement of the coil from the region in which the magnetic field is constant.

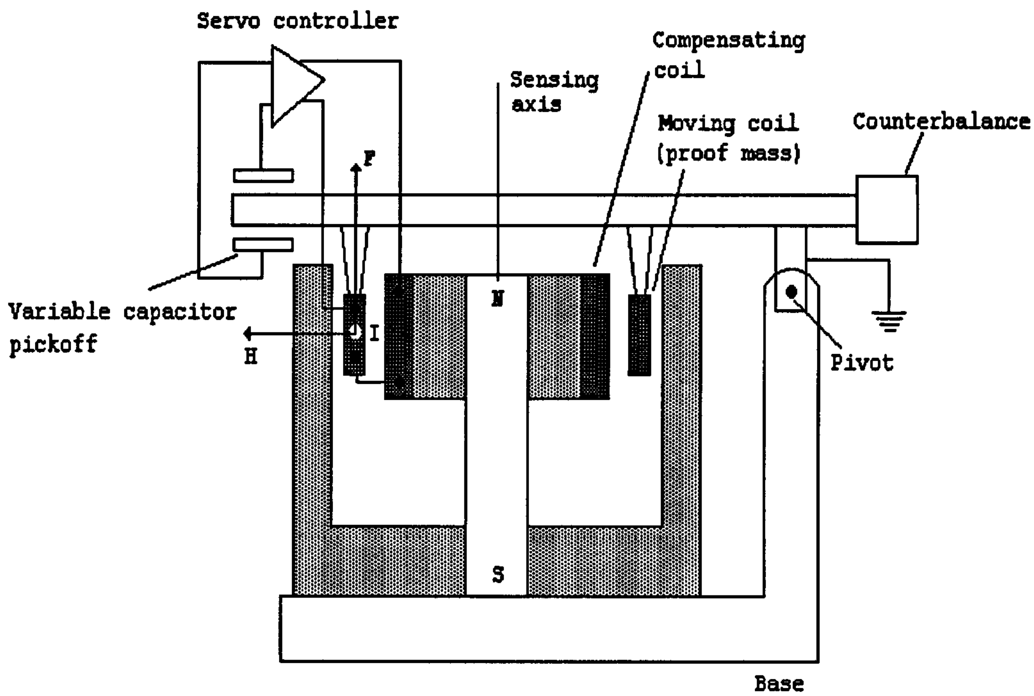

FIGURE 17.6 A basic coil and permanent magnet accelerometer. The coil is supported by an arm with minimum friction bearings to form a proof mass in a magnetic field. Displacement of the coil due to acceleration induces an electric potential in the coil to be sensed and processed. A servo system maintains the coil in a null position.

In these accelerometers, the magnetic structure must be shielded adequately to make the system insensitive to external disturbances or Earth's magnetic field. Also, in the presence of acceleration, there will be a temperature rise due to i^2R losses. The effect of these i^2R losses on the performance is determined by the thermal design and heat transfer properties of the accelerometer. In many applications, special care must be exercised in choosing the appropriate accelerometer such that the temperature rises caused by unexpected accelerations cannot affect excessively the scale factors or the bias conditions.

A simplified version of another type of servo-accelerometer is given in Figure 17.7. The acceleration a of the instrument case causes an inertial force F on the sensitive mass m, tending to make it pivot in its bearings or flexure mount. The rotation θ from neutral is sensed by an inductive pickup and amplified, demodulated, and filtered to produce a current i_a directly proportional to the motion from the null. This current is passed through a precision stable resistor R to produce the output voltage signal and is applied to a coil suspended in a magnetic field. The current through the coil produces magnetic torque on the coil, which takes action to return the mass to neutral. The current required to produce magnetic torque that just balances the inertial torque due to acceleration is directly proportional to acceleration a. Therefore, the output voltage e_0 becomes a measure of acceleration a. Since a nonzero displacement θ is necessary to produce the current i_a, the mass is not exactly returned to null, but becomes very close to zero because of the high gain amplifier. Analysis of the block diagram reveals that:

$$e_0/R = \left(mra - e_0 K_c/R\right) \times \left(K_p K_a/K_s\right) \Big/ \left(s^2/\omega^2{}_{nl} + 2\zeta_1 s/\omega_{nl} + 1\right) \tag{17.11}$$

Rearranging this expression gives:

$$mrRK_pK_a a/K_s = \left(s^2/\omega^2{}_{nl} + 2\zeta_1 s/\omega_{nl} + 1 + K_cK_pK_a a/K_s\right) e_0 \tag{17.12}$$

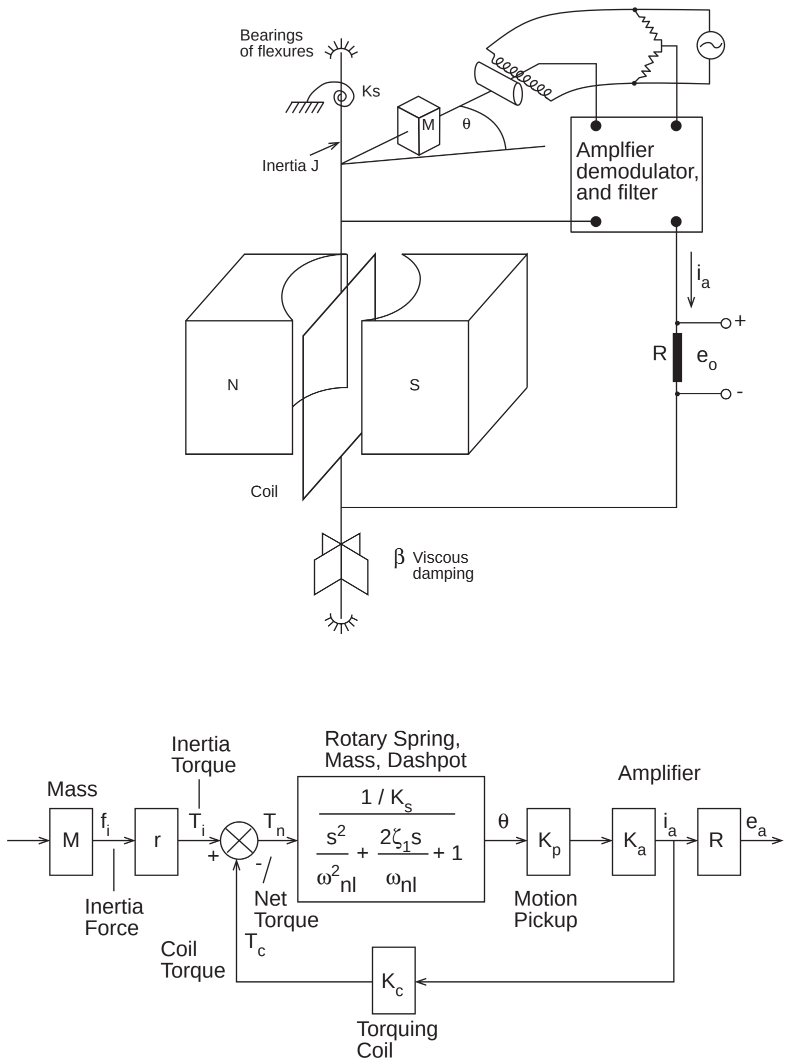

FIGURE 17.7 A simplified version of a rotational type servo-accelerometer. Acceleration of the instrument case causes an inertial force on the sensitive mass, tending to make it pivot in its bearings or flexure mount. The rotation from neutral is sensed by inductive sensing and amplified, demodulated, and filtered to produce a current directly proportional to the motion from the null. The block diagram representation is useful in analysis.

By designing the amplifier gain, K_a is made large enough so that $K_c K_p K_a a/K_s$ >> 1.0; then:

$$e_0/a(s) = K\Big/\left(s^2/\omega^2_{nl} + 2\zeta_1 s/\omega_{nl} + 1 + K_c K_p K_a a/K_s\right) e_0 \qquad (17.13)$$

where

$$K \cong MrR/K_c, \qquad \left(\text{V m}^{-1}\ \text{s}^{-2}\right) \qquad (17.14)$$

$$\omega_n \cong \omega_{nl}\sqrt{K_c K_p K_a/K_s} \qquad \text{rad/s} \qquad (17.15)$$

$$\zeta \cong \zeta_1 \Big/ \sqrt{K_c K_p K_a \big/ K_s} \tag{17.16}$$

Equation 17.14 shows that the sensitivity depends on the values of m, r, R, and K_c, all of which can be made constant. In this case, a high-gain feedback is useful in shifting the requirements for accuracy and stability from mechanical components to a selected few parameters where the requirements can easily be met. As in all feedback systems, the gain cannot be made arbitrarily high because of dynamic instability; however, a sufficiently high gain can be achieved to obtain good performance. An excellent comprehensive treatment of this topic is given by Doebelin, 1990; interested readers should refer to [3].

Induction Type Accelerometers

The cross-product relationship of current, magnetic field, and force gives the basis for induction type electromagnetic accelerometers, which are essentially generators rather than motors. One type of instrument, cup-and-magnet, includes a pendulous element with a pick-off and a servo-controller driving a tachometer coupling and a permanent magnet and a flux return ring, closely spaced with respect to an electrically conductive cylinder attached to the pendulus element. A rate proportional drag-force is obtained by electromagnetic induction effects between magnet and conductor. The pick-off senses pendulum deflection under acceleration and causes the servo-controller to turn the rotor in a sense to drag the pendulus element toward null. Under steady-state conditions, motor speed is a measure of the acceleration acting on the instrument. Stable servo operation is achieved employing a time-lead network to compensate the inertial time lag of the motor and magnet combination. The accuracy of servo type accelerometers is ultimately limited by consistency and stability of the scale factors of coupling devices and magnet-and-cup as a function of time and temperature.

Another accelerometer based on induction types is the *eddy current induction torque generation.* It was noted that the force-generating mechanism of an induction accelerometer consists of a stable magnetic field, usually supplied by a permanent magnet, which penetrates orthogonally through a uniform conduction sheet. The movement of the conducting sheet relative to the magnetic field in response to an acceleration results in a generated electromotive potential in each circuit in the conductor. This action is in accordance with the law of Faraday's principle. In induction-type accelerometers, the induced eddy currents are confined to the conductor sheet, making the system essentially a drag coupling.

A typical commercial instrument based on the servo-accelerometer principle might have a micromachined quartz flexure suspension, differential capacitance angle pick-off, air squeeze film, plus servo lead compensation for system damping. Of the various available models, a 30 g range unit has threshold and resolution of 1 μg, frequency response flat within 0.05% at 10 Hz and 2% at 100 Hz, natural frequency 500 Hz, damping ratio 0.3 to 0.8, and transverse or cross-axis sensitivity 0.1%. If, for example, the output current is about 1.3 mA g^{-1} and a 250 Ω readout resistor would give about ±10 V full scale at 30 g. These accelerometers are good with respect to precision, and are used in many applications, such as aircraft and missile control systems, the measurement of tilt angles, axle angular bending in weight and balance systems, etc.

17.3 Piezoelectric Accelerometers

Piezoelectric accelerometers are used widely for general-purpose acceleration, shock, and vibration measurements. They basically are motion transducers with large output signals and comparatively small sizes. They are available with very high natural frequencies and are therefore suitable for high-frequency applications and shock measurements.

These devices utilize a mass in direct contact with the piezoelectric component, or crystal, as shown in Figure 17.8. When a varying motion is applied to the accelerometer, the crystal experiences a varying force excitation ($F = ma$), causing a proportional electric charge q to be developed across it.

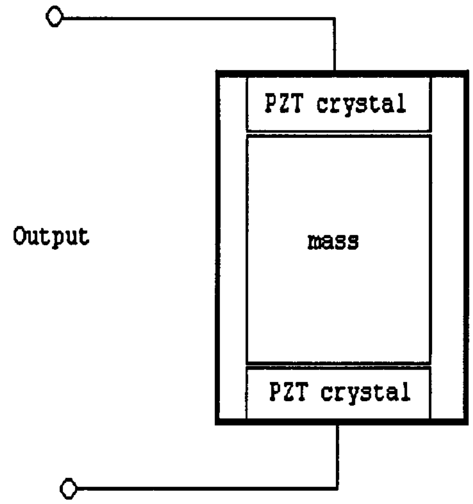

FIGURE 17.8 A compression-type piezoelectric accelerometer. The crystals are under compression at all times, either by a mass or mass and spring arrangement. Acceleration causes a deformation of the crystal, thus producing a proportional electric signal. They are small in size and widely used. They demonstrate poor performance at low frequencies.

$$q = d_{ij} F = d_{ij} m a \tag{17.17}$$

where q = the charge developed
d_{ij} = the material's piezoelectric coefficient

As Equation 17.17 shows, the output from the piezoelectric material is dependent on its mechanical properties, d_{ij}. Two commonly used piezoelectric crystals are lead-zirconate titanate ceramic (PZT) and quartz. They are both self-generating materials and produce a large electric charge for their size. The piezoelectric strain constant of PZT is about 150 times that of quartz. As a result, PZTs are much more sensitive and smaller in size than their quartz counterparts. In the accelerometers, the mechanical spring constants for the piezoelectric components are high, and the inertial masses attached to them are small. Therefore, these accelerometers are useful for high frequency applications. Figure 17.9 illustrates a typical frequency response for a PZT device. Since piezoelectric accelerometers have comparatively low mechanical impedances, their effects on the motion of most structures is negligible. They are also manufactured to be rugged and they have outputs that are stable with time and environment.

Mathematically, their transfer function approximates to a third-order system as:

$$e_0(s)/a(s) = \left(K_q / C\omega_n^2\right)\tau s \Big/ \left[\left(\tau s + 1\right)\left(s^2/\omega_n^2 + 2\zeta s/\omega_n + 1\right)\right] \tag{17.18}$$

where K_q = the piezoelectric constant related to charge (C cm)
τ = the time constant of the crystal

It is worth noting that the crystal itself does not have a time constant τ, but the time constant is observed when the accelerometer is connected into an electric circuit (e.g., an *RC* circuit).

The low-frequency response is limited by the piezoelectric characteristic $\tau s/(\tau s + 1)$, while the high-frequency response is related to mechanical response. The damping factor ζ is very small, usually less than 0.01 or near zero. Accurate low-frequency response requires large τ, which is usually achieved by

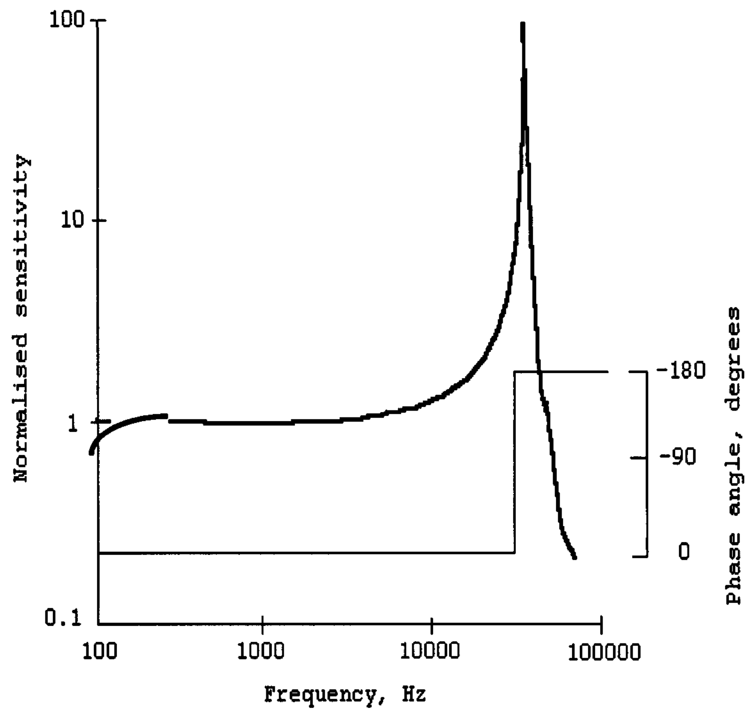

FIGURE 17.9 Frequency response of a typical piezoelectric accelerometer. Measurements are normally confined to the linear portion of the response curve. The upper frequency of the accelerometer is limited by the resonance of the PZT crystal. The phase angle is constant up to the resonance frequency.

the use of high-impedance voltage amplifiers. At very low frequencies, thermal effects can have severe influences on the operation characteristics.

In piezoelectric accelerometers, two basic design configurations are used: compression types and shear stress types. In compression-type accelerometers, the crystal is held in compression by a preload element; therefore, the vibration varies the stress in compressed mode. In the shear accelerometer, vibration simply deforms the crystal in shear mode. The compression type has a relatively good mass/sensitivity ratio and hence exhibits better performance. But, since the housing acts as an integral part of the spring mass system, it may produce spurious interfaces in the accelerometer output, if excited in its proper natural frequency.

Microelectronic circuits have allowed the design of piezoelectric accelerometers with charge amplifiers and other signal conditioning circuits built into the instrument housing. This arrangement allows greater sensitivity, high-frequency response, and smaller size accelerometers, thus lowering the initial and implementation costs.

Piezoelectric accelerometers are available in a wide range of specifications and are offered by a large number of manufacturers. For example, the specifications of a shock accelerometer may have 0.004 pC g^{-1} in sensitivity and a natural frequency of up to 250,000 Hz, while a unit designed for low-level seismic measurements might have 1000 pC g^{-1} in sensitivity and only 7000 Hz natural frequency. They are manufactured as small as 3×3 mm in dimensions with about 0.5 g in mass, including cables. They have excellent temperature ranges, and some of them are designed to survive intensive radiation environment of nuclear reactors. However, piezoelectric accelerometers tend to have larger cross-axis sensitivity than other types; about 2% to 4%. In some cases, large cross-axis sensitivity can be used during installation for the correct orientation of the device. These accelerometers can be mounted with threaded studs, with cement or wax adhesives, or with magnetic holders.

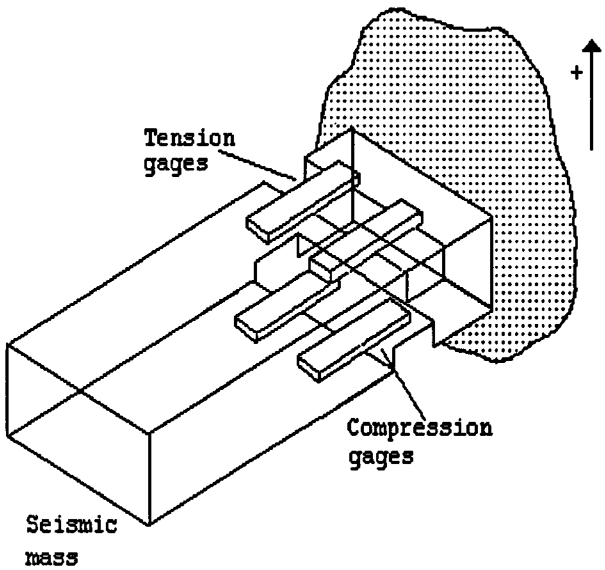

FIGURE 17.10 Bonding of piezoelectric and piezoresistive elements onto an inertial system. As the inertial member vibrates, deformation of the tension and compression gages causes the resistance to change. The change in resistance is picked up and processed further. Accelerometers based on PZTs are particularly useful in medium- to high-frequency applications.

17.4 Piezoresistive Accelerometers

Piezoresistive accelerometers are essentially semiconductor strain gages with large gage factors. High gage factors are obtained because the material resistivity is dependent primarily on the stress, not only on dimensions. The increased sensitivity is critical in vibration measurement because it allows the miniaturization of the accelerometer. Most piezoresistive accelerometers use two or four active gages arranged in a Wheatstone bridge. Extra-precision resistors are used, as part of the circuit, in series with the input to control the sensitivity, balancing, and offsetting temperature effects. The mechanical construction of a piezoresistive accelerometer is shown in Figure 17.10.

In some applications, overload stops are necessary to protect the gages from high-amplitude inputs. These instruments are useful for acquiring vibration information at low frequencies (e.g., below 1 Hz). In fact, the piezoresistive sensors are inherently true static acceleration measurement devices. Typical characteristics of piezoresistive accelerometers may be 100 mV g^{-1} in sensitivity, 0 to 750 Hz in frequency range, 2500 Hz in resonance frequency, 25 g in amplitude range, 2000 g in shock rating, 0 to 95°C in temperature range, with a total mass of about 25 g.

17.5 Differential-Capacitance Accelerometers

Differential-capacitance accelerometers are based on the principle of change of capacitance in proportion to applied acceleration. They come in different shapes and sizes. In one type, the seismic mass of the accelerometer is made as the movable element of an electrical oscillator as shown in Figure 17.11. The seismic mass is supported by a resilient parallel-motion beam arrangement from the base. The system is characterized to have a certain defined nominal frequency when undisturbed. If the instrument is accelerated, the frequency varies above and below the nominal value, depending on the direction of acceleration.

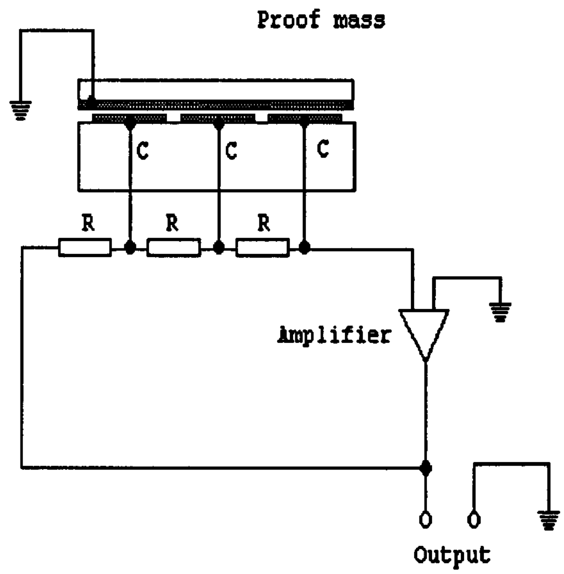

FIGURE 17.11 A typical differential capacitive accelerometer. The proof mass is constrained in its null position by a spring. Under acceleration, variable frequencies are obtained in the electric circuit. In a slightly different version, the proof mass may be constrained by an electrostatic-feedback-force, thus resulting in convenient mechanical simplicity.

The seismic mass carries an electrode located in opposition to a number of base-fixed electrodes that define variable capacitors. The base-fixed electrodes are resistance coupled in the feedback path of a wide-band, phase-inverting amplifier. The gain of the amplifier is made of such a value to ensure maintenance of oscillations over the range of variation of capacitance determined by the applied acceleration. The value of the capacitance C for each of the variable capacitor is given by:

$$C = \varepsilon kS/h \tag{17.19}$$

where k = dielectric constant
ε = capacitivity of free space
S = area of electrode
h = variable gap

Denoting the magnitude of the gap for zero acceleration as h_0, the value of h in the presence of acceleration a may be written as:

$$h = h_0 + ma/K \tag{17.20}$$

where m = the value of the proof mass and K is the spring constant. Thus,

$$C = \varepsilon kS/\left(h_0 + ma/K\right) \tag{17.21}$$

If, for example, the frequency of oscillation of the resistance-capacitance type circuit is given by the expression:

$$f = \sqrt{6}/2\pi RC \tag{17.22}$$

Substituting this value of C in Equation 17.21 gives:

$$f = \left(h_0 + ma/K\right) \sqrt{6}/2\pi R\varepsilon kS \tag{17.23}$$

Denoting the constant quantity ($\sqrt{6}/2\pi R\varepsilon kS$) as B and rewriting Equation 17.23 gives:

$$f = Bh_0 + Bma/K \tag{17.24}$$

The first term on the right-hand side expresses the fixed bias frequency f_0, and the second term denotes the change in frequency resulting from acceleration, so that the expression may be written as:

$$f = f_0 + f_a \tag{17.25}$$

If the output frequency is compared with an independent source of constant frequency f_0, f_a can be determined.

A commonly used example of a capacitive-type accelerometer is based on a thin diaphragm with spiral flexures that provide the spring, proof mass, and moving plate necessary for the differential capacitor, as shown in Figure 17.12. Plate motion between the electrodes pumps air parallel to the plate surface

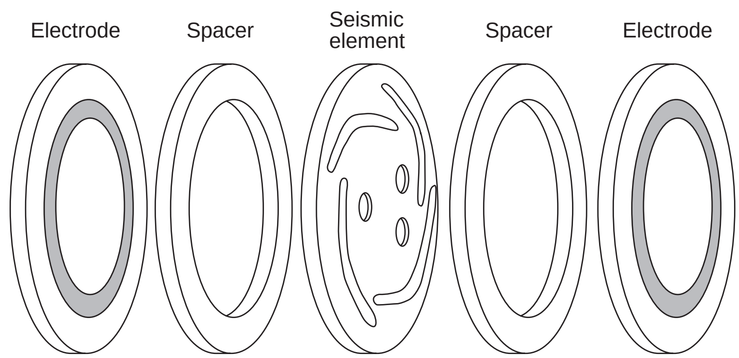

FIGURE 17.12 Diaphragm-type capacitive accelerometer. The seismic element is cushioned between the electrodes. Motion of the mass between the electrodes causes air movement passing through the holes, which provides a squeeze film damping. In some cases, oil can be used as the damping element.

and through holes in the plate to provide squeeze film damping. Since air viscosity is less temperature sensitive than oil, the desired damping ratio of 0.7 hardly changes more than 15%. A family of such instruments are readily available, having full-scale ranges from ±0.2 g (4 Hz flat response) to ±1000 g (3000 Hz), cross-axis sensitivity less than 1%, and full-scale output of ±1.5 V. The size of a typical device is about 25 mm^3 with a mass of 50 g.

17.6 Strain-Gage Accelerometers

Strain gage accelerometers are based on resistance properties of electrical conductors. If a conductor is stretched or compressed, its resistance alters due to two reasons: dimensional changes and the changes in the fundamental property of material called *piezoresistance*. This indicates that the resistivity ρ of the conductor depends on the mechanical strain applied onto it. The dependence is expressed as the gage factor

$$\left(\mathrm{d}R/R\right)/\left(\mathrm{d}L/L\right) = 1 + 2\nu + \left(\mathrm{d}\rho/\rho\right)/\left(\mathrm{d}L/L\right) \tag{17.26}$$

where
1 = resistance change due to length
2ν = resistance change due to area
$(\mathrm{d}\rho/\rho)/(\mathrm{d}L/L)$ = resistance change due to piezoresistivity

In acceleration measurements, the resistance strain gages can be selected from different types, including unbonded metal-wire gages, bonded metal-wire gages, bonded metal-foil gages, vacuum-deposited thin-metal-film gages, bonded semiconductor gages, diffused semiconductor gages, etc. But, usually, bonded and unbonded metal-wire gages find wider applications in accelerometers. Occasionally, bonded semiconductor gages, known as piezoresistive transducers, are used but suffer from high-temperature sensitivities, nonlinearity, and some mounting difficulties. Nevertheless, in recent years, they have found new application areas with the development of micromachine transducer technology, which is discussed in detail in the micro-accelerometer section.

Unbonded-strain-gage accelerometers use the strain wires as the spring element and as the motion transducer, using similar arrangements as in Figure 17.10. They are useful for general-purpose motion and vibration measurements from low to medium frequencies. They are available in wide ranges and characteristics, typically ±5 g to ±200 g full scale, natural frequency 17 Hz to 800 Hz, 10 V excitation voltage ac or dc, full-scale output ±20 mV to ±50 mV, resolution less than 0.1%, inaccuracy less than 1% full scale, and cross-axis sensitivity less than 2%. Their damping ratio (using silicone oil damping) is 0.6 to 0.8 at room temperature. These instruments are small and lightweight, usually with a mass of less than 25 g.

Bonded-strain-gage accelerometers generally use a mass supported by a thin flexure beam. The strain gages are cemented onto the beam to achieve maximum sensitivity, temperature compensation, and sensitivity to both cross-axis and angular accelerations. Their characteristics are similar to unbonded-strain gage accelerometers, but have larger sizes and weights. Often, silicone oil is used for damping. Semiconductor strain gages are widely used as strain sensors in cantilever-beam/mass types of accelerometers. They allow high outputs (0.2 V to 0.5 V full scale). Typically, a ±25 g acceleration unit has a flat response from 0 Hz to 750 Hz, a damping ratio of 0.7, a mass of 28 g, and an operational temperature of –18°C to +93°C. A triaxial ±20,000 g model has flat response from 0 kHz to 15 kHz, a damping ratio 0.01, a compensation temperature range of 0°C to 45°C, 13 × 10 × 13 mm in size, and 10 g in mass.

17.7 Seismic Accelerometers

These accelerometers make use of a seismic mass that is suspended by a spring or a lever inside a rigid frame. The schematic diagram of a typical instrument is shown in Figure 17.1. The frame carrying the seismic mass is connected firmly to the vibrating source whose characteristics are to be measured. As the system vibrates, the mass tends to remain fixed in its position so that the motion can be registered as a relative displacement between the mass and the frame. This displacement is sensed by an appropriate transducer and the output signal is processed further. Nevertheless, the seismic mass does not remain absolutely steady; but for selected frequencies, it can satisfactorily act as a reference position.

By proper selection of mass, spring, and damper combinations, the seismic instruments may be used for either acceleration or displacement measurements. In general, a large mass and soft spring are suitable for vibration and displacement measurements, while a relatively small mass and stiff spring are used in accelerometers.

The following equation may be written by using Newton's second law of motion to describe the response of seismic arrangements similar to shown in Figure 17.1.

$$m\mathrm{d}^2 x_2/\mathrm{d}t^2 + c\mathrm{d}x_2/\mathrm{d}t + kx_2 = c\mathrm{d}x_1/\mathrm{d}t + kx_1 + mg\cos(\theta) \qquad (17.27)$$

where x_1 = the displacement of the vibration frame
x_2 = the displacement of the seismic mass
c = velocity constant
k = spring constant

Taking $m\mathrm{d}^2x_1/\mathrm{d}t^2$ from both sides of the equation and rearranging gives:

$$m\mathrm{d}^2 z/\mathrm{d}t^2 + c\,\mathrm{d}z/\mathrm{d}t + kz = mg\cos(\theta) - m\mathrm{d}^2 x_1/\mathrm{d}t^2 \qquad (17.28)$$

where $z = x_2 - x_1$ is the relative motion between the mass and the base
θ = the angle between sense axis and gravity

In Equation 17.27, it is assumed that the damping force on the seismic mass is proportional to velocity only. If a harmonic vibratory motion is impressed on the instrument such that:

$$x_1 = x_0 \sin\omega_1 t \qquad (17.29)$$

where ω_1 is the frequency of vibration, in rad s^{-1}.

Writing

$$m\mathrm{d}^2 x_1/dt^2 = m\, x_0\, \omega_1^2 \sin\omega_1 t$$

modifies Equation 17.28 as:

$$m\mathrm{d}^2 z/\mathrm{d}t^2 + c\,\mathrm{d}z/\mathrm{d}t + kz = mg\cos(\theta) + m\, a_1 \sin\omega_1 t \qquad (17.30)$$

where $a_1 = m\, x_0\, \omega_1^2$

Equation 17.30 will have transient and steady-state solutions. The steady-state solution of the differential Equation 17.30 can be determined as:

$$z = \left[mg\cos(\theta)/k\right] + \left[m\, a_1 \sin\omega_1 t/\left(k - m\,\omega_1^2 + jc\omega_1\right)\right] \qquad (17.31)$$

Rearranging Equation 17.31 results in:

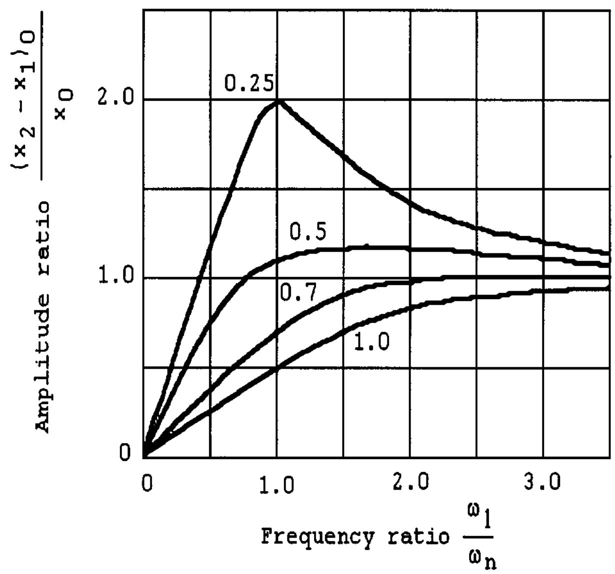

FIGURE 17.13 A typical displacement of a seismic instrument. Amplitude becomes large at low damping ratios. The instrument constants should be selected such that, in measurements, the frequency of vibration is much higher than the natural frequency (e.g., greater than 2). Optimum results are obtained when the value of instrument constant c/c_c is about 0.7.

$$z = \left[mg\cos(\theta)/\omega_m\right] + \left\{a_1 \sin(\omega_1 - \phi)\Big/\left[\omega_m^2\left(1-r^2\right)^2 + \left(2\zeta r\right)^2\right]^{1/2}\right\} \tag{17.32}$$

where $\omega_n = \sqrt{k/m}$ = the natural frequency of the seismic mass

ζ = $c/2\sqrt{km}$ = the damping ratio, also can be written in terms of critical damping ratio as $\zeta = c/c_c$, where ($c_c = 2\sqrt{km}$)

ϕ = $\tan^{-1}(c\omega_1/(k - m\omega_1^2))$ = the phase angle

r = ω_1/ω_m = the frequency ratio

A plot of Equation 17.32, $(x_1 - x_2)_0/x_0$ against frequency ratio ω_1/ω_n, is illustrated in Figure 17.13. This figure shows that the output amplitude is equal to the input amplitude when $c/c_c = 0.7$ and $\omega_1/\omega_n > 2$. The output becomes essentially a linear function of the input at high frequency ratios. For satisfactory system performance, the instrument constant c/c_c and ω_n should carefully be calculated or obtained from calibrations. In this way, the anticipated accuracy of measurement can be predicted for frequencies of interest. A comprehensive treatment of the analysis is given by McConnell [1].

If the seismic instrument has a low natural frequency and a displacement sensor is used to measure the relative motion z, then the output is proportional to the displacement of the transducer case. If the velocity sensor is used to measure the relative motion, the signal is proportional to the velocity of the transducer. This is valid for frequencies significantly above the natural frequency of the transducer. However, if the instrument has a high natural frequency and the displacement sensor is used, the measured output is proportional to the acceleration:

$$kz = m\,\mathrm{d}^2x_1/\mathrm{d}t^2 \tag{17.33}$$

This equation is true since displacement x_2 becomes negligible in comparison to x_1.

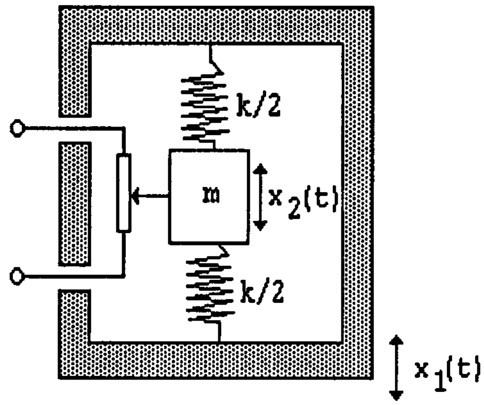

FIGURE 17.14 A potentiometer accelerometer. The relative displacement of the seismic mass is sensed by a potentiometer arrangement. The potentiometer adds extra weight, making these accelerometers relatively heavier. Suitable liquids filling the frame can be used as damping elements. These accelerometers are used in low-frequency applications.

In these instruments, the input acceleration a_0 can be calculated by simply measuring $(x_1 - x_2)_0$. Generally, in acceleration measurements, unsatisfactory performance is observed at frequency ratios above 0.4. Thus, in such applications, the frequency of acceleration must be kept well below the natural frequency of the instrument. This can be accomplished by constructing the instrument to have a low natural frequency by selecting soft springs and large masses.

Seismic instruments are constructed in a variety of ways. Figure 17.14 illustrates the use of a voltage divider potentiometer for sensing the relative displacement between the frame and the seismic mass. In the majority of potentiometric instruments, the device is filled with a viscous liquid that interacts continuously with the frame and the seismic mass to provide damping. These accelerometers have low frequency of operation (less than 100 Hz) and are mainly intended for slow varying acceleration and low-frequency vibrations. A typical family of such instrument offers many different models, covering the range of ±1 g to ±50 g full scale. The natural frequency ranges from 12 Hz to 89 Hz, and the damping ratio ζ can be kept between 0.5 to 0.8 using a temperature-compensated liquid damping arrangement. Potentiometer resistance can be selected in the range of 1000 Ω to 10,000 Ω, with corresponding resolution of 0.45% to 0.25% of full scale. The cross-axis sensitivity is less than ±1%. The overall accuracy is ±1% of full scale or less at room temperatures. The size is about 50 mm^3; with a mass of about 1/2 kg.

Linear variable differential transformers (LVDT) offer another convenient means to measure the relative displacement between the seismic mass and the accelerometer housing. These devices have higher natural frequencies than potentiometer devices, up to 300 Hz. Since the LVDT has lower resistance to the motion, it offers much better resolution. A typical family of liquid-damped differential-transformer accelerometers exhibits the following characteristics: full scale range from ±2 g to ±700 g, natural frequency from 35 Hz to 620 Hz, nonlinearity 1% of full scale, the full scale output is about 1 V with an LVDT excitation of 10 V at 2000 Hz, damping ratio 0.6 to 0.7, residual voltage at null is less than 1%, and hysteresis less than 1% full scale; the size is 50 mm^3, with a mass of about 120 g.

Electric resistance strain gages are also used for displacement sensing of the seismic mass as shown in Figure 17.15. In this case, the seismic mass is mounted on a cantilever beam rather than on springs. Resistance strain gages are bonded on each side of the beam to sense the strain in the beam resulting from the vibrational displacement of the mass. Damping for the system is provided by a viscous liquid that entirely fills the housing. The output of the strain gages is connected to an appropriate bridge circuit. The natural frequency of such a system is about 300 Hz. The low natural frequency is due to the need for a sufficiently large cantilever beam to accommodate the mounting of the strain gages. Other types of seismic instruments using piezoelectric transducers and seismic masses are discussed in detail in the section dedicated to piezoelectric-type accelerometers.

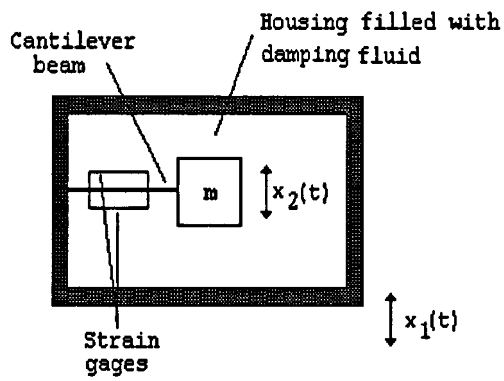

FIGURE 17.15 A strain gage seismic instrument. The displacement of the proof mass is sensed by piezoresistive strain gages. The natural frequency of the system is low, due to the need of a long lever beam to accommodate strain gages. The signal is processed by bridge circuits.

Seismic vibration instruments are affected seriously by the temperature changes. Devices employing variable resistance displacement sensors will require correction factors to account for resistance change due to temperature. The damping of the instrument may also be affected by changes in the viscosity of the fluid due to temperature. For example, the viscosity of silicone oil, often used in these instruments, is strongly dependent on temperature. One way of eliminating the temperature effect is by using an electrical resistance heater in the fluid to maintain the temperature at a constant value regardless of surrounding temperatures.

17.8 Inertial Types, Cantilever, and Suspended-Mass Configuration

There are a number of different inertial accelerometers, including gyropendulum, reaction-rotor, vibrating string, and centrifugal-force-balance types. In many of them, the force required to constrain the mass in the presence of the acceleration is supplied by an inertial system.

A vibrating string type instrument, Figure 17.16, makes use of proof mass supported longitudinally by a pair of tensioned, transversely vibrating strings with uniform cross-section, and equal lengths and masses. The frequency of vibration of the strings is set to several thousand cycles per second. The proof mass is supported radially in such a way that the acceleration normal to strings does not affect the string tension. In the presence of acceleration along the sensing axis, a differential tension exists on the two strings, thus altering the frequency of vibration. From the second law of motion, the frequencies can be written as:

$$f_1^{\,2} = T_1 \Big/ \left(4 m_s l\right) \quad \text{and} \quad f_2^{\,2} = T_2 \Big/ \left(4 m_s l\right) \tag{17.34}$$

where T is the tension, and m_s and l are the masses and lengths of strings, respectively.

Quantity $(T_1 - T_2)$ is proportional to ma, where a is the acceleration along the axis of the strings. An expression for the difference of the frequency-squared terms may be written as:

$$f_1^{\,2} - f_2^{\,2} = \left(T_1 - T_2\right) \Big/ \left(4 m_s l\right) = ma \Big/ \left(4 m_s l\right) \tag{17.35}$$

Hence,

$$f_1 - f_2 = ma \Big/ \left[\left(f_1 + f_2\right) 4 m_s l\right] \tag{17.36}$$

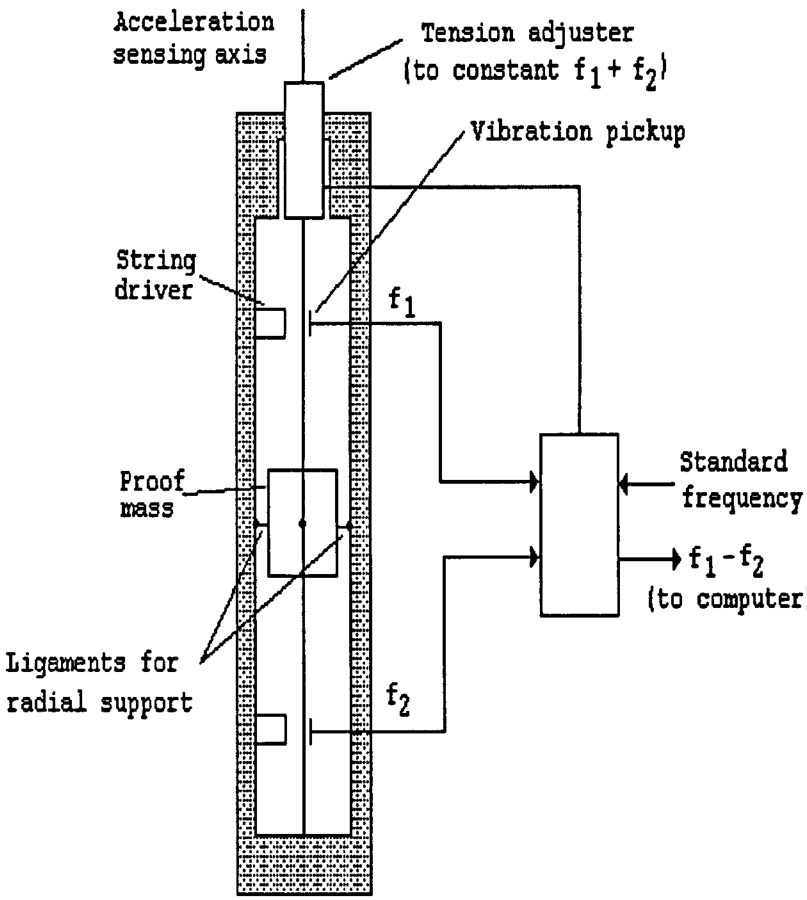

FIGURE 17.16 A vibrating string accelerometer. A proof mass is attached to two strings of equal mass and length and supported radially by suitable bearings. The vibration frequencies of strings are dependent on the tension imposed by the acceleration of the system in the direction of the sensing axis.

The sum of frequencies ($f_1 + f_2$) can be held constant by servoing the tension in the strings with reference to the frequency of a standard oscillator. Then, the difference between the frequencies becomes linearly proportional to acceleration. In some versions, the beam-like property of the vibratory elements is used by gripping them at nodal points corresponding to the fundamental mode of vibration of the beam. Improved versions of these devices lead to cantilever-type accelerometers, as discussed next.

In cantilever-type accelerometers, a small cantilever beam mounted on the block is placed against the vibrating surface, and an appropriate mechanism is provided for varying the beam length. The beam length is adjusted such that its natural frequency is equal to the frequency of the vibrating surface — hence the resonance condition obtained. Slight variations of cantilever beam-type arrangements are finding new applications in microaccelerometers.

In a different type suspended mass configuration, a pendulum is used that is pivoted to a shaft rotating about a vertical axis. Pick-offs are provided for the pendulum and the shaft speed. The system is servo-controlled to maintain it at null position. Gravitational acceleration is balanced by the centrifugal acceleration. Shaft speed is proportional to the square root of the local value of the acceleration.

All inertial force accelerometers described above have the property of absolute instruments. That is, their scale factors can be predetermined solely by establishing mass, length, and time quantities, as distinguished from voltage, spring stiffness, etc.

17.9 Electrostatic Force Feedback Accelerometers

Electrostatic accelerometers are based on Coulomb's law between two charged electrodes. They measure the voltage in terms of force required to sustain a movable electrode of known area, mass, and separation from an affixed electrode. The field between the electrodes is given by:

$$E = Q/\varepsilon kS \tag{17.37}$$

where E is the intensity or potential gradient (dV/dx), Q is charge, S area of the conductor, and k is the dielectric constant of the space outside the conductor.

Using this expression, it can be shown that the force per unit area of the charged conductor (in N m^{-2}) is given by:

$$F/S = Q^2/\left(2\varepsilon kS^2\right) = \varepsilon kE^2/2 \tag{17.38}$$

In an electrostatic-force-feedback-type accelerometer (similar in structure to that in Figure 17.10), an electrode of mass m and area S is mounted on a light pivoted arm for moving relative to the fixed electrodes. The nominal gap, h, between the pivoted and fixed electrodes is maintained by means of a force balancing servo system capable of varying the electrode potential in response to signals from a pick-off that senses relative changes in the gaps.

Considering one movable electrode and one stationary electrode, and assuming that the movable electrode is maintained at a bias potential V_1 and the stationary one at a potential V_2, the electrical intensity E in the gap can be expressed as:

$$E_1 = \left(V_1 - V_2\right)/h \tag{17.39}$$

so that the force of attraction may be found as:

$$F_1 = \varepsilon kE^2S/\left(2h^2\right) = \varepsilon k\left(V_1 - V_2\right)^2 S/\left(2h^2\right) \tag{17.40}$$

In the presence of acceleration, if V_2 is adjusted to restrain the movable electrode to null position; the expression relating acceleration and electrical potential may be given by:

$$a = F_1/m = \varepsilon k\left(V_1 - V_2\right)^2 S/\left(2h^2m\right) \tag{17.41}$$

The device thus far described can measure acceleration in one direction only, and the output is of quadratic character; that is:

$$\left(V_1 - V_2\right) = D\sqrt{a} \tag{17.42}$$

where D = the constant of proportionality.

The output can be linearized in a number of ways; for example, by quarter-square method. If the servo controller applies a potential $-V_2$ to other fixed electrode, the force of attraction between this electrode and the movable electrode becomes:

$$a = F_1/m = \varepsilon k\left(V_1 + V_2\right)^2 S/\left(2h^2m\right) \tag{17.43}$$

and the force balance equation of the movable electrode when the instrument experiences a downward acceleration a is:

$$ma = F_1 - F_2 = \left[\left(V_1 + V_2\right)^2 - \left(V_1 - V_2\right)^2\right]\varepsilon kS \Big/ \left(2h^2 m\right)$$

or

$$= \varepsilon kS\left(4V_1V_2\right) \Big/ \left(2h^2 m\right) \tag{17.44}$$

Hence, if the bias potential V_1 is held constant and the gain of the control loop is high so that variations in the gap are negligible, the acceleration becomes a linear function of the controller output voltage V_2 as:

$$a = V_2\left[\varepsilon kS\left(2V_1\right) \Big/ \left(h^2 m\right)\right] \tag{17.45}$$

The principal difficulty in mechanizing the electrostatic force accelerometer is the relatively high electric field intensity required to obtain adequate force. Also, extremely good bearings are necessary. Damping can be provided electrically, or by viscosity of gaseous atmosphere in the interelectrode space if the gap h is sufficiently small.

The main advantages of the electrostatic accelerometers include extreme mechanical simplicity, low power requirements, absence of inherent sources of hysteresis errors, zero temperature coefficients, and ease of shielding from stray fields.

17.10 Microaccelerometers

By the end of the 1970s, it became apparent that the essentially planar processing IC (integrated circuit) technology could be modified to fabricate three-dimensional electromechanical structures, called micromachining. Accelerometers and pressure sensors were among the first IC sensors. The first accelerometer was developed in 1979. Since then, technology has been progressing steadily to the point where an extremely diverse range of accelerometers is readily available. Most sensors use bulk micromachining rather than surface micromachining techniques. In bulk micromachining, the flexures, resonant beams, and all other critical components of the accelerometer are made from bulk silicon in order to exploit the full mechanical properties of single-crystal silicon. With proper design and film process, bulk micromachining yields an extremely stable and robust accelerometer.

The selective etching of multiple layers of deposited thin films, or surface micromachining, allows movable microstructures to be fabricated on silicon wafers. With surface micromachining, layers of structure material are disposed and patterned, as shown in Figure 17.17. These structures are formed by polysilicon and a sacrificial material such as silicon dioxide. The sacrificial material acts as an intermediate spacer layer and is etched away to produce a free-standing structure. Surface machining technology also allows smaller and more complex structures to be built in multiple layers on a single substrate.

The operational principles of microaccelerometers are very similar to capacitive force-balance-type accelerometers or vibrating beam types, as discussed earlier. Nevertheless, manufacturing techniques may change from one manufacturer to another. In general, vibrating beam accelerometers are preferred because of better air gap properties and improved bias performance characteristics.

The vibrating beam accelerometers, also called resonant beam force transducers, are made in such a way that an acceleration along a positive input axis places the vibrating beam in tension. Thus, the resonant frequency of the vibrating beam increases or decreases with the applied acceleration. A mechanically coupled beam structure, also known as a double-ended tuning fork (DETF), is shown in Figure 17.18.

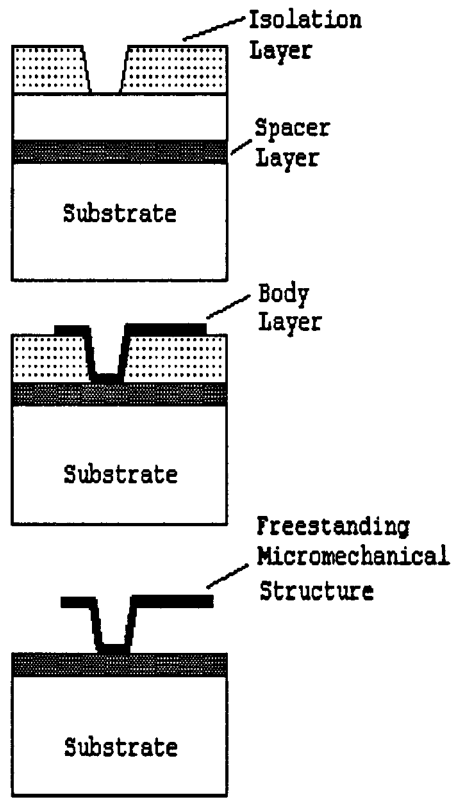

FIGURE 17.17 Steps of surface micromachining. The acceleration-sensitive, three-dimensional structure is formed on a substrate and a sacrificial element. The sacrificial element is etched to leave a free-standing structure. The spacing between the structure and substrate is about 2 μm.

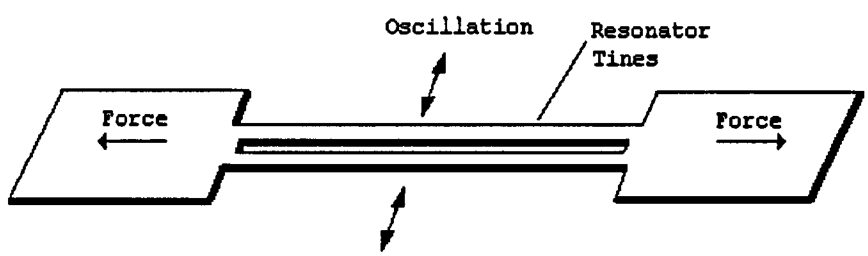

FIGURE 17.18 A double-ended tuning fork (DETF) acceleration transducer. Two beams are vibrated 180° out of phase to eliminate reaction forces at the beam ends. The resonant frequency of the beam is altered by acceleration. The signal processing circuits are also integrated in the same chip.

In DETF, an electronic oscillator capacitively couples energy into two vibrating beams to keep them oscillating at their resonant frequency. The beams vibrate 180° out of phase to cancel reaction forces at the ends. The dynamic cancellation effect of the DETF design prevents energy from being lost through the ends of the beam. Hence, the dynamically balanced DETF resonator has a high *Q* factor, which leads to a stable oscillator circuit. The acceleration signal is an output from the oscillator as a frequency-modulated square wave that can be used for digital interface.

The frequency of resonance of the system must be much higher than any input acceleration and this limits the measurable range. In a typical military micromachine accelerometer, the following characteristics are given: range ±1200 g, sensitivity 1.11 Hz g^{-1}, bandwidth 2500 Hz, unloaded DETF frequency 9952 Hz, frequency at +1200 g is 11221 Hz, frequency at –1200 g is 8544 Hz, the temperature sensitivity 5 mg °C. Accelerometer size is 6 mm diameter × 4.3 mm length, with a mass of about 9 g, and it has a turn on time less then 60 s. The accelerometer is powered with +9 to +16 V dc and the nominal output is a 9000 Hz square wave.

Surface micromachining has also been used to manufacture specific application accelerometers, such as air-bag applications in automotive industry. In one type, a three-layer differential capacitor is created by alternate layers of polysilicon and phosphosilicate glass (PSG) on a 0.38 mm thick and 100 mm long wafer. A silicon wafer serves as the substrate for the mechanical structure. The trampoline-shaped middle layer is suspended by four supporting arms. This movable structure is the seismic mass for the accelerometer. The upper and lower polysilicon layers are fixed plates for the differential capacitors. The glass is sacrificially etched by hydrofluoric acid.

17.11 Cross-Axis Sensitivity

A vibrational structure may have been subjected to different forms of vibrations, such as compressional, torsional, transverse, etc.; or a combination of all these vibrations may take place simultaneously, which makes the analysis and measurements difficult and complex. It was discussed earlier that the differential equations governing the vibrational motion of a structure were dependent on the number of degrees of freedom, which can be described as a function of the space coordinates $f(x,y,z,t)$. For example, the transverse vibrations of structures may be a fourth-order differential equation.

Fortunately, most common acceleration and vibration measurements are simple in nature, being either compressional or torsional types. They can easily be expressed as second-order differential equations, as explained in the frequency response section. However, during measurements, most accelerometers are affected by transverse vibrations and their sensitivity can play a major role in the accuracy of the measurements.

The transverse, also known as cross-axis sensitivity, of an accelerometer is its response to acceleration in a plane perpendicular to the main accelerometer axis, as shown in Figure 17.19. The cross-axis sensitivity is normally expressed in percent of the main axis sensitivity and should be as low as possible. There is not a single value of cross-axis sensitivity, but it varies depending on the direction. The direction of minimum sensitivity is usually supplied by the manufacturer.

The measurement of the maximum cross-axis sensitivity is part of the individual calibration procedure and should always be less than 3% to 4%. If high levels of transverse vibration are present, this may result in erroneous overall results. In this case, separate arrangements should be made to establish the level and frequency contents of the cross-axis vibrations. Cross-axis sensitivities of typical accelerometers are mentioned in the relevant sections: 2% to 3% for piezoelectric types and less than 1% in most others.

17.12 Selection, Full-Scale Range, and Overload Capability

Ultimate care must be exercised for the selection of the correct accelerometer to meet the requirements of a particular application. At first glance, there may seem to be a confusingly large repertoire of accelerometers available; however, they can be classified into two main groups. The first group are the general-purpose accelerometers offered in various sensitivities, frequencies, full scale, and overload ranges, with different mechanical and electrical connection options. The second group of accelerometers are the special types that have characteristics targeted toward a particular application.

In deciding the application type (e.g., general purpose or special) and the accelerometer to be employed, the following characteristics need to be considered: transient response or cross-axis sensitivity; frequency range; sensitivity, mass and dynamic range; cross-axis response; and environmental conditions such as temperature, cable noise, etc. Some useful hints about these characteristics are given below.

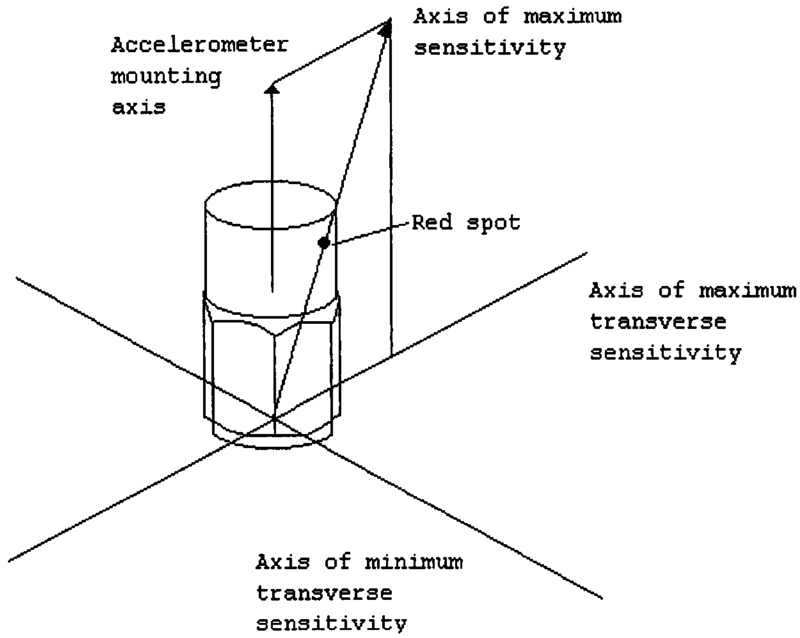

FIGURE 17.19 Vectorial illustration of cross-axis sensitivity. Accelerometers may sense vibrations not only in the direction of main axis, but also perpendicular to the main axis. These cross-axis responses are minimized in many accelerometers to a value less than 1%. Sometimes, this sensitivity is used to determine the correct orientation of the device.

The Frequency Range

Acceleration measurements are normally confined to using the linear portion of the response curve. The response is limited at the low frequencies as well as at the high frequencies by the natural resonances. As a rule of thumb, the upper frequency limit for the measurement can be set to one third of the accelerometer's resonance frequency such that the vibrations measured will be less than 1 dB in linearity. It should be noted that an accelerometer's useful frequency range is significantly higher, that is, to 1/2 or 2/3 of its resonant frequency. The measurement frequencies may be set to higher values in applications where lower linearity (e.g., 3 dB) may be acceptable, as in the case of monitoring internal conditions of machines since the reputability is more important than the linearity. The lower measuring frequency limit is determined by two factors. The first is the low-frequency cut-off of the associated preamplifiers. The second is the effect of ambient temperature fluctuations to which the accelerometer could be sensitive.

The Sensitivity, Mass, and Dynamic Range

Ideally, the higher the transducer sensitivity, the better; but compromises might have to be made for sensitivity versus frequency, range, overload capacity, and size.

Accelerometer mass becomes important when using it on small and light test objects. The accelerometer should not load the structural member, since additional mass can significantly change the levels and frequency presence at measuring points and invalidate the results. As a general rule, the accelerometer mass should not be greater than one tenth the effective mass of the part or the structure that is mounted onto for measurements.

The dynamic range of the accelerometer should match the high or low acceleration levels of the measured objects. General-purpose accelerometers can be linear up to 5000 g to 10,000 g, which is well into the range of most mechanical shocks. Special accelerometers can measure up to 100,000 g.

An important point in the practical application of accelerometers is that if mechanical damping is a problem, air damping is preferable to oil damping, since oil damping is extremely sensitive to viscosity changes. If the elements are temperature stable, electronic damping may be sufficient.

The Transient Response

Shocks are characterized as sudden releases of energy in the form short-duration pulses exhibiting various shapes and rise times. They have high magnitudes and wide frequency contents. In applications where transients and shock measurements are involved, the overall linearity of the measuring system may be limited to high and low frequencies by phenomena known as *zero shift* and *ringing*, respectively. The zero shift is caused by both the phase nonlinearity in the preamplifiers and the accelerometer not returning to steady-state operation conditions after being subjected to high shocks. Ringing is caused by high-frequency components of the excitation near resonance frequency preventing the accelerometer to return back to its steady-state operation condition. To avoid measuring errors due to these effects, the operational frequency of the measuring system should be limited to the linear range.

Full-Scale Range and Overload Capability

Most accelerometers are able to measure acceleration in both positive and negative directions. They are also designed to be able to accommodate overload capacity. Appropriate discussions are made on full-scale range and overload capacity of accelerometers in the relevant sections. Manufacturers usually supply information on these two characteristics.

Environmental Conditions

In the selection and implementation of accelerometers, environmental conditions such as temperature ranges, temperature transients, cable noise, magnetic field effects, humidity, acoustic noise, etc. need to be considered. Manufacturers also supply information on environmental conditions.

17.13 Signal Conditioning

Common signal conditioners are appropriate for interfacing accelerometers to computers or other instruments for further signal processing. Caution needs to be exercised to provide appropriate electric load to self-generating accelerometers. Generally, the generated raw signals are amplified and filtered suitably by the circuits within the accelerometer casing supplied by manufacturers. Nevertheless, piezoelectric and piezoresistive transducers require special signal conditioners with certain characteristics, as discussed next. Examples of signal conditioning circuits are also given for microaccelerometers.

Signal Conditioning Piezoelectric Accelerometers

The piezoelectric accelerometer supplies a very small energy to the signal conditioner. It has a high capacitive source impedance. The equivalent circuit of a piezoelectric accelerometer can be regarded as an active capacitor that charges itself when loaded mechanically. The configuration of external signal conditioning elements is dependent on the equivalent circuit selected. The charge amplifier design of the conditioning circuit is the most common approach since the system gain and low-frequency responses are well defined. The performance of the circuit is independent of cable length and capacitance of the accelerometer.

The charge amplifier consists of a charge converter output voltage that occurs as a result of the charge input signal returning through the feedback capacitor to maintain the input voltage at the input level close to zero, as shown in Figure 17.20. An important point about charge amplifiers is that their sensitivities can be standardized. They basically convert the input charge to voltage first and then amplify this voltage. With the help of basic operational-type feedback, the amplifier input is maintained at essentially

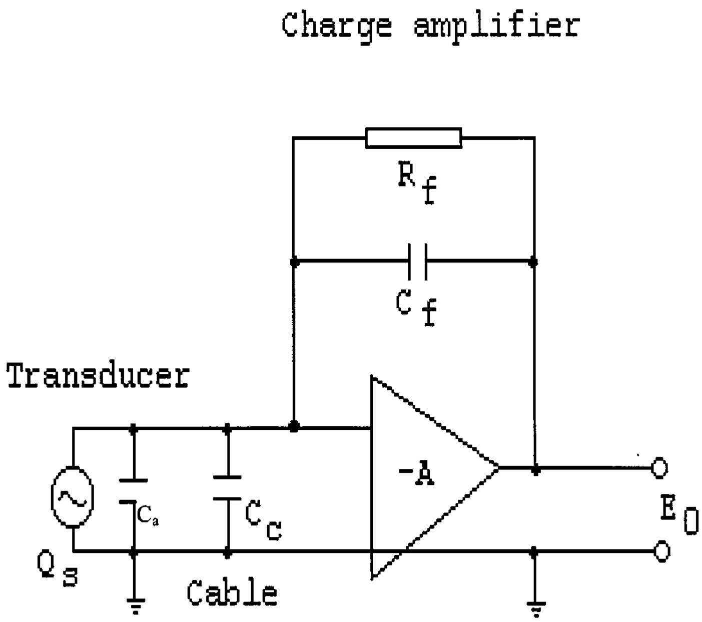

FIGURE 17.20 A typical charge amplifier. The transducer charge, which is proportional to acceleration, is first converted to voltage form to be amplified. The output voltage is a function of the input charge. The response of the amplifier can be approximated by a first-order system. In PZT transducers, the preamplifier is integrated within the same casing.

zero volts; therefore, it looks like a short circuit to the input. The charge converter output voltage that occurs as a result of a charge input signal is returned through the feedback capacitor to maintain the voltage at the input level near zero. Thus, the charge input is stored in the feedback capacitor, producing a voltage across it, that is equal to the value of the charge input divided by the capacitance of the feedback capacitor. The complete transfer function of the circuit describing the relationship between the output voltage and the input acceleration magnitude can be determined by:

$$E_o / a_0 = S_a jR_f C_f \omega \Big/ \left\{1 + jR_f C_f \left[1 + \left(C_a + C_c\right) \Big/ \left(1 + G\right) \times C_f\right] \omega\right\} \tag{17.46}$$

where E_o = charge converter output (V)
a_0 = magnitude of acceleration (m s^{-2})
S_a = accelerometer sensitivity (mV g^{-1})
C_a = accelerometer capacitance (F)
C_c = cable capacitance (F)
C_f = feedback capacitance (F)
R_f = feedback loop resistance
G = amplifier open loop gain

In most applications, since C_f is selected to be large compared to $(C_a + C_c)/(1 + G)$, the system gain becomes independent of the cable length. In this case, the denominator of the equation can be simplified to give a first-order system with roll-off at:

$$f_{-3\text{ dB}} = 1 \Big/ \left(2\pi R_f C_f\right) \tag{17.47}$$

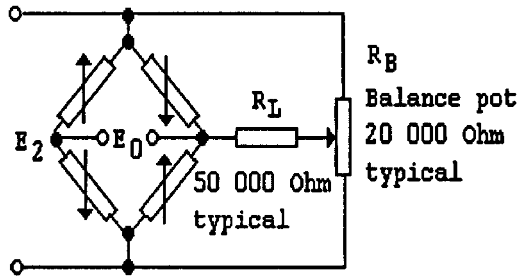

FIGURE 17.21 Bridge circuit for piezoresistive and strain gage accelerometers. The strain gages form the four arms of the bridge. The two extra resistors are used for balancing and fine adjustment purposes. This type of arrangement reduces temperature effects.

with a slope of 10 dB per decade. For practical purposes, the low-frequency response of this system is a function of well-defined electronic components and does not vary with cable length. This is an important feature when measuring low-frequency vibrations.

Many accelerometers are manufactured with preamplifiers and other signal-conditioning circuits integrated with the transducer enclosed in the same casing. Some accelerometer preamplifiers include integrators to convert the acceleration proportional outputs to either velocity or displacement proportional signals. To attenuate noise and vibration signals that lie outside the frequency range of interest, most preamplifiers are equipped with a range of high-pass and low-pass filters. This avoids interference from electric noise or signals inside the linear portion of the accelerometer frequency range. Nevertheless, it is worth mentioning that these devices usually have two time constants, external and internal. The mixture of these two time constants can lead to problems, particularly at low frequencies. The internal time constant is usually fixed by the manufacturer in design and construction. Special care must be observed to take care of the effect of external time constants in many applications by mainly observing impedance matching.

Signal Conditioning of Piezoresistive Transducers

Piezoresistive transducers generally have high amplitude outputs, low output impedances, and low intrinsic noise. Most of these transducers are designed for constant voltage excitations. They are usually calibrated for constant current excitations to make them independent of external influences. Many piezoresistive transducers are configured as full-bridge devices. Some have four active piezoresistive arms and, together with two fixed precision resistors permit shunt calibration in the signal conditioner, as shown in Figure 17.21.

Microaccelerometers

In microaccelerometers, signal conditioning circuitry is integrated within the same chip with the sensor as shown in Figure 17.22. A typical example of the signal conditioning circuitry is given in Figure 17.23 in block diagram form. In this type of accelerometer, the electronic system is essentially a crystal-controlled oscillator circuit and the output signal of the oscillator is a frequency modulated acceleration signal. Some circuits provide a buffered square-wave output that can directly be interfaced digitally. In this case, the need for analog to digital conversion is eliminated, thus removing one of the major sources of error. In other types of accelerometers, signal conditioning circuits such as analog to digital converters (ADC) are retained within the chip.

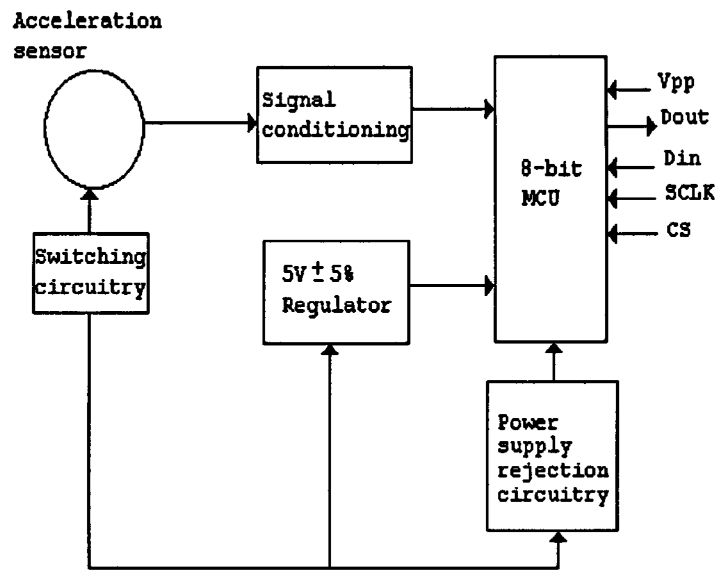

FIGURE 17.22 A block diagram of an accelerometer combined with MCU. The signal conditioning, switching, and power supply circuits are integrated to form a microaccelerometer. The device can directly be interfaced with a digital signal processor or a computer. In some cases, ADCs and memory are also integrated.

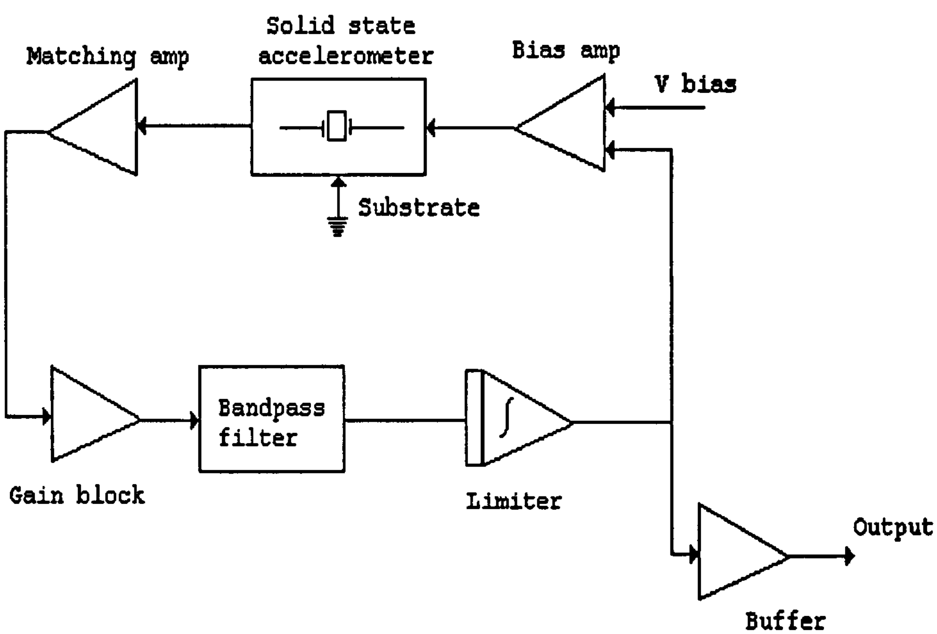

FIGURE 17.23 Block diagram of a signal-conditioning circuit of a microaccelerometer. The output signal of the oscillator is a frequency-modulated acceleration signal. The circuit provides a buffered square-wave frequency output that can be read directly into a digital device.

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List of Manufacturers

Allied Signal, Inc.
101 Colombia Road
Dept. CAC
Morristown, NJ 07962

Bokam Engineering, Inc.
9552 Smoke Tree Avenue
Fountain Valley, CA 92708
Tel: (714) 962-3121
Fax: (714) 962-5002

CEC Vibration Products
Division of Sensortronics
196 University Parkway
Pomona, CA 91768
Tel: (909) 468-1345 or
(800) 468-1345
Fax: (909) 468-1346

Dytran Instrument, Inc.
Dynamic Transducers and Systems
21592 Marilla Street
Chatsworth, CA 91311
Tel: (800) 899-7818
Fax: (800) 899-7088

Endevco
30700 Rancho Viejo Road
San Juan Capistrona, CA 92675
Tel: (800) 289-8204
Fax: (714) 661-7231

Entran Devices, Inc.
10-T Washington Avenue
Fairfield, NJ 07004
Tel: (800) 635-0650

First Inertia Switch
G-10386 N. Holly Road
Dept. 10, PO Box 704
Grand Blanc, MI 48439
Tel: (810) 695-8333 or
(800) 543-0081
Fax: (810) 695-0589

Instrumented Sensor Technology
4701 A Moor Street
Okemos, MI 48864
Tel: (517) 349-8487
Fax: (517) 349-8469

Jewel Electrical Instruments
124 Joliette Street
Manchester, NH 03102
Tel: (603) 669-6400 or
(800) 227-5955
Fax: (603) 669-5962

Kistler Instruments Company
75 John Glenn Drive
Amherst, NY 14228-2171
Tel: (800) 755-5745

Lucas Control Production, Inc.
1000 Lucas Way
Hampton, VA 23666
Tel: (800) 745-8008
Fax: (800) 745-8004

Metrix Instrument Company
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(800) 867-3890
Fax: (614) 486-0506

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Acton, MA 01720
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(800) 257-3872
Fax: (508) 264-0292

Silicon Microstructures, Inc.
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Fremond, CA 94358
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Fax: (510) 490-1119

SKF Condition Monitoring
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San Diego, CA 92123
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Fax: (619) 496-3531

Summit Instruments, Inc.
2236 N. Cleveland-Massillon Road
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Fax: (216) 659-3286

Wilcoxon Research
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Gaithersburg, MD 20878
Tel: (800) 842-7367
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